Original Research Article

Nonlocal Schrödinger–Maxwell System Involving Fractional p-Laplacian With Singular Nonlinearity

Asymptotic Analysis
1–40

DOI: 10.1177/09217134251343511
journals.sagepub.com/home/asy

Abdelhamid Gouasmia[1,2] and Youssef El Hadfi[3]

Abstract

In this paper, we study the following nonlocal system exhibiting singular nonlinearity and weighted singular terms:

$$\begin{cases} (-\Delta)_p^s u + v\,u^{p-1} = h(x)u^{-\alpha}, & u > 0 \quad \text{in } \Omega; \quad u = 0, \quad \text{in } \mathbb{R}^N \setminus \Omega, \\ (-\Delta)_p^s v = u^p, & v > 0 \quad \text{in } \Omega; \quad v = 0, \quad \text{in } \mathbb{R}^N \setminus \Omega, \end{cases}$$

where $0 < s < 1$, $p > 1$, $\alpha > 0$, and $\Omega \subset \mathbb{R}^N$, with $N > sp$, is an open bounded domain with $C^{1,1}$ boundary $\partial\Omega$. The function $h : \Omega \to \mathbb{R}^+$ exhibits growth of negative powers of the distance function $d(x) := \text{dist}(x, \partial\Omega)$ near the boundary, that is, $h(x) \sim d^{-\beta}(x)$ for some $\beta \geq 0$, when x is close to the boundary $\partial\Omega$. For $\beta < sp$, we discuss the existence of a positive weak solution $(u, v) \in W^{s,p}_{\text{loc}}(\Omega) \times W^{s,p}_{\text{loc}}(\Omega)$ using the classical method of regularization and the fixed point theorem together. Indeed, we found some essential uniform a priori estimates for the approximating sequence before proceeding to the limits. Moreover, we address the uniqueness of finite energy solutions, that is, $(u, v) \in W^{s,p}_0(\Omega) \times W^{s,p}_0(\Omega)$, and demonstrate that this solution pair is a saddle point of a suitable functional when $\alpha < 1$. We also provide the boundary behavior of the weak solutions in terms of the distance function. Finally, we establish the nonexistence of a weak solution for the case where $\beta \geq sp$.

Keywords

Schrödinger–Maxwell system, fractional p-Laplace operator, singular nonlinearity, existence and non-existence, uniqueness

1 Introduction and Main Results

In this work, we investigate the existence, nonexistence, uniqueness, and the boundary behavior of the weak solution to the following nonlocal Schrödinger–Maxwell system:

$$\begin{cases} (-\Delta)_p^s u + v\,u^{p-1} = \dfrac{h(x)}{u^\alpha}, & u > 0 \quad \text{in } \Omega; \quad u = 0, \quad \text{in } \mathbb{R}^N \setminus \Omega, \\ (-\Delta)_p^s v = u^p, & v > 0 \quad \text{in } \Omega; \quad v = 0, \quad \text{in } \mathbb{R}^N \setminus \Omega. \end{cases} \tag{S}$$

[1]Department of Mathematics, Faculty of Sciences and Technology, Mohamed Cherif Messaadia University, Souk Ahras, Algeria
[2]Laboratoire d'equations aux dérivées partielles non linéaires et histoire des mathématiques, Ecole Normale Supérieure, Vieux Kouba, Algiers
[3]Laboratory LIPIM, National School of Applied Sciences Khouribga, Sultan Moulay Slimane University, Khouribga, Morocco

Corresponding Author:
Youssef El Hadfi, Laboratory LIPIM, National School of Applied Sciences Khouribga, Sultan Moulay Slimane University, Khouribga, 25000, Morocco.
Email: yelhadfi@gmail.com