# Model Reference Tracking Control for Uncertain Takagi-Sugeno Systems subject to Sensor Faults


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## Objective and main contribution

### Objective

Design an active fault tolerant tracking controller strategy to preserve the closed-loop system stability in spite of sensor faults and uncertainties

### Contribution

- Uncertain nonlinear systems represented by T-S models (Unmeasurable premise variables, Unknown bounded disturbances)
- Active fault tolerant tracking control law based on the estimated states
- Descriptor observer design to estimate sensor faults and system states

# Outline

1. Objective and main contribution
2. Takagi-Sugeno approach for modeling
3. Fault tolerant tracking control strategy
4. Illustrative example
5. Conclusions and perspectives

## Takagi-Sugeno approach for modeling

- The Takagi-Sugeno model structure is given by :

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\left(A_i x(t) + B_i u(t)\right) \\ y(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\, C_i x(t) \end{cases}$$

$x(t) \in \mathbb{R}^n$ is the system state variable, $u(t) \in \mathbb{R}^m$ is the control input and $y(t) \in \mathbb{R}^p$ the system output

- Interpolation mechanism

$$\sum_{i=1}^{r} \mu_i(\xi(t)) = 1 \; et \; \mu_i(\xi(t)) \geq 0, \forall t, \forall i \in \{1 \cdots r\}$$

- Obtaining a Takagi-Sugeno model

✓ *Identification*

✓ *linéarisation Approach*

✓ *Nonlinear sector Approach*

## Takagi-Sugeno approach for modeling

- Faulty uncertain system (Sensor faults)

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) \left(\mathbb{A}_i x(t) + \mathbb{B}_i u(t)\right) + B_d d(t) \\ y(t) = Cx(t) + D_f f(t) \end{cases}$$

*f(t): sensor fault vector*

*d(t) : unknown bounded disturbance vector*

$\mathbb{A}_i = A_i + \Delta A_i$

$\mathbb{B}_i = B_i + \Delta B_i$

$\Delta A_i = M_a \gamma(t) N_{ai}$ , $\Delta B_i = M_b \gamma(t) N_{bi}$

$\gamma(t)\gamma^T(t) \leq I$

$I$, being the identity matrix, $M_{a,b}$ and $N_{(a,b)i}$ are known real constant matrices of appropriate dimensions.

- $M$ represents the maximum percentage of the state matrices variation
- $N$ are matrices including the nominal values.

## Takagi-Sugeno approach for modeling

### ❑ Descriptor faulty uncertain system (Sensor faults)

$$\begin{cases} \bar{E}\dot{\bar{x}}(t) = \sum_{i=1}^{r} h_i(\xi(t))\left(\bar{\mathbb{A}}_i \bar{x}(t) + \bar{\mathbb{B}}_i u(t)\right) + \bar{B}_f d(t) + \bar{D}_h g(t) \\ y(t) = \bar{C}\bar{x}(t) = C_0 \bar{x}(t) + g(t) \end{cases}$$

where

$$\bar{x}(t) = \begin{bmatrix} x(t) \\ g(t) \end{bmatrix} \in \mathbb{R}^{n+p}, \quad \bar{E} = \begin{bmatrix} I_n & 0 \\ 0 & 0 \end{bmatrix}, \quad g(t) = D_f f(t) \in \mathbb{R}^p$$

with

$$\bar{\mathbb{A}}_i = \bar{A}_i + \overline{\Delta A_i} = \begin{bmatrix} A_i & 0 \\ 0 & -I_p \end{bmatrix} + \begin{bmatrix} \Delta A_i & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{\mathbb{B}}_i = \bar{B}_i + \overline{\Delta B_i} = \begin{bmatrix} B_i \\ 0 \end{bmatrix} + \begin{bmatrix} \Delta B_i \\ 0 \end{bmatrix}$$

### ❑ Objectives

1. *Adopt a T-S descriptor system representation approach to ensure the estimation of both the state and sensor fault vectors.*
2. *Design a fault tolerant controller to make the uncertain faulty system states follow as closely as possible the model reference states.*

## Fault tolerant tracking control strategy

❑ Reference model

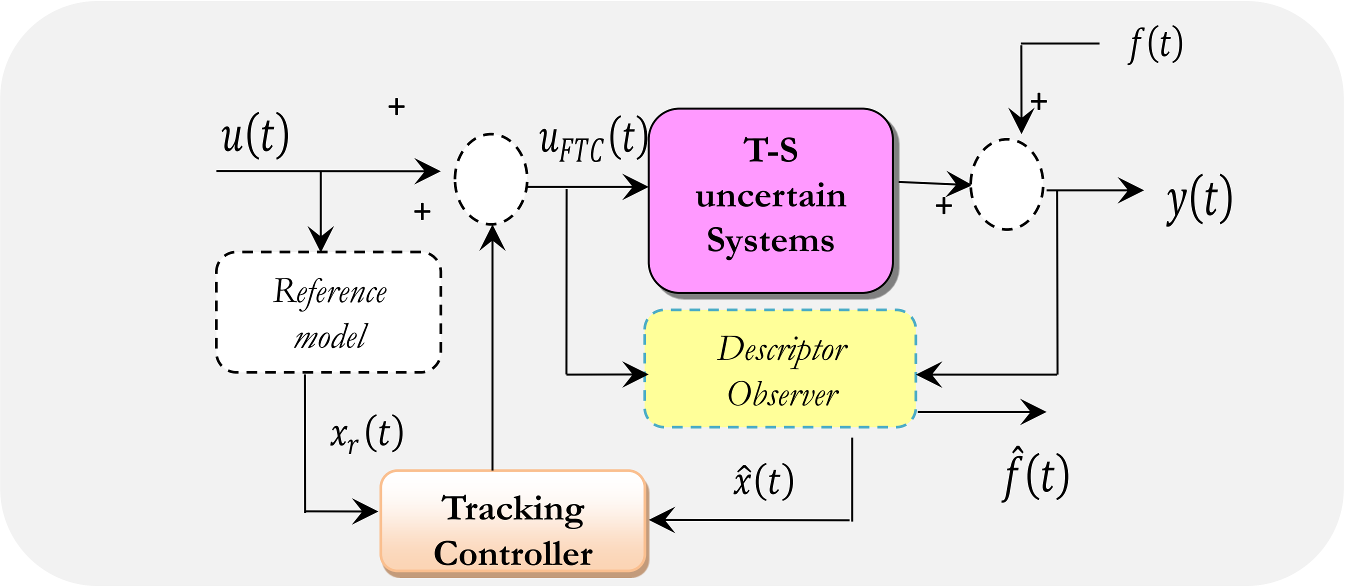
$$\begin{cases} \dot{x}_r(t) = \sum_{i=1}^{r} h_i(\xi(t)) \left(A_i x_r(t) + B_i u(t)\right) \\ y(t) = C x_r(t) \end{cases}$$

❑ Control law

$$u_{FTC}(t) = u(t) + \sum_{j=1}^{r} h_j\left(\hat{\xi}(t)\right)\left(K_j\left(\hat{x}(t) - x_r(t)\right)\right)$$

✓ $K_i \in \mathbb{R}^{m \times n}$ are the state-feedback gain matrices to be determine

✓ The FT tracking control law is chosen now as a classical PDC law, but based on the knowledge of “fault free’’ estimated states

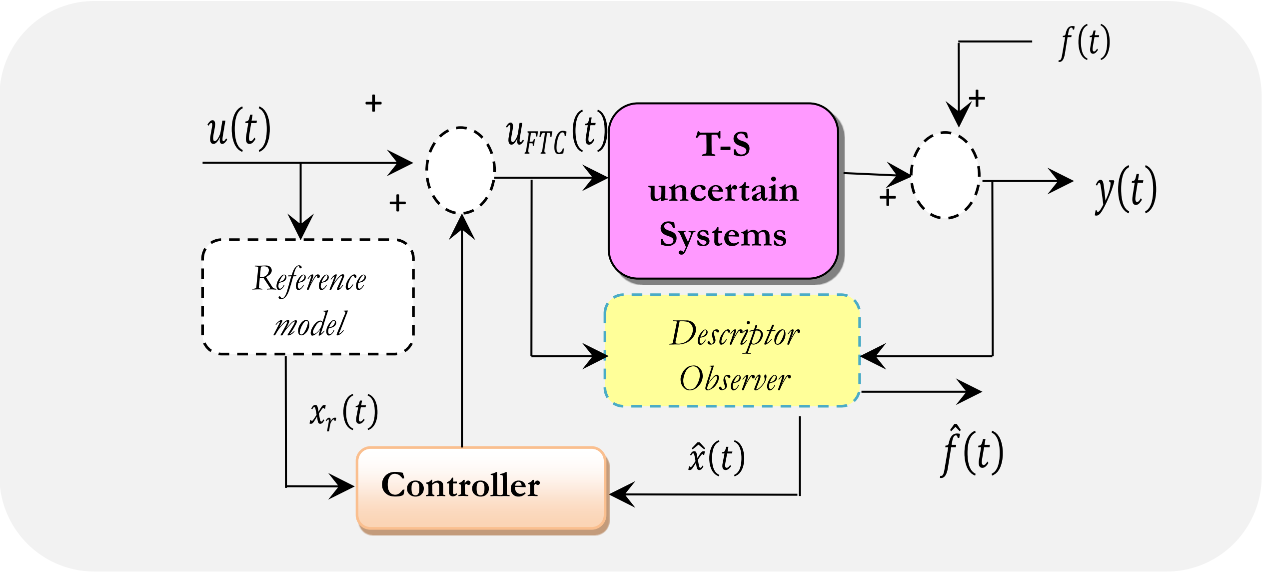


## Fault tolerant tracking control strategy

- State descriptor observer

$$\begin{cases} E\dot{z}(t) = \sum_{j=1}^{r} h_j(\hat{\xi})\left(F_j z(t) + \bar{B}_j u_{FTC}(t)\right) \\ \hat{\bar{x}}(t) = z(t) + Ly(t) \\ \hat{y}(t) = C_0\hat{\bar{x}}(t) = C\hat{x}(t) \end{cases}$$

- ✓ $z(t) \in \mathbb{R}^{n+p}$ is the auxiliary state vector of the observer
- ✓ $\hat{\xi}(t)$ is the unmeasured premise variable depending partially or completely on the estimated state $\hat{x}(t)$.
- ✓ $F_i, E$ and $L$ are the observer gains to be determined.



## Fault tolerant tracking control strategy

### ❑ Observer and controller gain design

✓ Define the state estimation error and state tracking error signals as:

$$\begin{pmatrix} e_t(t) \\ e_s(t) \end{pmatrix} = \begin{pmatrix} x(t) - x_r(t) \\ \bar{x}(t) - \hat{\bar{x}}(t) \end{pmatrix}$$

✓ Dynamic of the state estimation error and state tracking error

$$\dot{e}_s(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi(t))h_j\left(\hat{\xi}(t)\right)\left[S_{ij}\, e_s(t) + T e_t(t) + \mathfrak{C}_{ij}\, x(t) + \mathfrak{H} d(t) + Q u(t)\,\right]$$

$$\dot{e}_t(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi(t))h_j\left(\hat{\xi}(t)\right)\left(\Delta A_i + \mathbb{B}_i K_j\right)e_t(t) + \Delta B_i u(t)$$

with

$$F_j = \begin{bmatrix} A_j & 0 \\ -C & -I_p \end{bmatrix},\ L = \begin{bmatrix} 0 \\ I_p \end{bmatrix},\ E = \begin{bmatrix} I_n + \Theta C & \Theta \\ RC & R \end{bmatrix}$$

$\Theta \in \mathbb{R}^{p\times p}$ and $R \in \mathbb{R}^{n\times p}$ are free matrices to be determined which are chosen to ensure the non singularity of matrix $E$.

## Fault tolerant tracking control strategy

### ❑ Observer and controller gains design

✓ Defining the augmented state vector

$$X^T(t) = [e_t^T(t) \;\; e_s^T(t) \;\; x^T(t)]$$

✓ The following closed-loop system is obtained

$$\dot{X}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi(t))h_j\left(\hat{\xi}(t)\right) \times \{\mathfrak{S}_{ij}\,[X(t)] + \mathfrak{F}_i \cdot [\Upsilon(t)]\}$$

where

$$\mathfrak{S}_{ij} = \begin{pmatrix} (\Delta A_i + \mathbb{B}_i K_j) & 0 & 0 \\ T & S_{ij} & \mathfrak{C}_{ij} \\ \mathbb{B}_i K_j & -\mathbb{B}_i \widetilde{K}_j & \mathbb{A}_i \end{pmatrix} \qquad \mathfrak{F}_i = \begin{pmatrix} \Delta B_i & 0 \\ V & \mathfrak{H} \\ \mathbb{B}_i & B_f \end{pmatrix}$$

and

$$\Upsilon(t) = (u(t) \quad d(t))^T$$

## Fault tolerant tracking control strategy

### ❑ Stability analysis

✓Consider the quadratic Lyapunov function

$$V(\boldsymbol{X}(\boldsymbol{t})) = \boldsymbol{X}^T(t)\mathbb{P}\boldsymbol{X}(\boldsymbol{t}) \quad \mathbb{P} = \mathbb{P}^T = diag[P_1 \quad P_2 \quad P_3]$$

- Error convergence to 0 ( absence of disturbances)
- $\mathcal{L}_2$ Constraint

$$\int_0^t \boldsymbol{X}^{\boldsymbol{T}}(\tau)Q\,\boldsymbol{X}(\boldsymbol{\tau})\,d\tau \leq \eta^2 \int_0^t \Upsilon^T(\tau)\Upsilon(\tau)\,d\tau$$

Where $\eta$ represents the attenuation level and $Q = diag(I \ \ I \ \ 0)$

### ❑Conditions

✓The tracking and the estimation errors must therefore satisfy the following inequality:

$$\begin{cases} min \quad \eta \\ \boldsymbol{X}^T(t)\Im(\mathbb{P}^T\mathfrak{S}_{ij})\boldsymbol{X}(\boldsymbol{t}) + \Im(\boldsymbol{X}^T(t)\mathbb{P}\mathfrak{F}_i\Upsilon(t)) + \boldsymbol{X}^T(t)Q\boldsymbol{X}(\boldsymbol{t}) - \eta^2\Upsilon^T(t)\Upsilon(t) < 0 \end{cases}$$

The main idea for the development is to separate the constant and the time-varying parts in $\mathfrak{S}_{ij}(t)$ and $\mathfrak{F}_i(t)$

## Fault tolerant tracking control strategy

### ❑ Theorem

*The uncertain T-S system is asymptotically stable via the fault tolerant tracking controller if there exist symmetric definite positive matrices* $P_1, P_{21}, P_{22}, P_3$*, matrices* $W_1, W_2$ *and the scalars* $\lambda_1^b, \lambda_3^b, \lambda_4^b, \lambda_5^b, \lambda_6^b, \lambda_1^a, \lambda_2^a, \lambda_3^a$ *such that the following LMI conditions are satisfied for all* $i, j = 1,2\cdots, r$ *and* $i \neq j$*:*

Minimize $\eta > 0$ such that:

$$\Psi_{ii} < 0$$

$$\frac{2}{r-1}\Psi_{ii} + \Psi_{ij} + \Psi_{ji} < 0 \quad \text{where} \quad \Psi_{ij} = \begin{pmatrix} \Psi_{ij}^{11} + \Xi_{\mathrm{ij}} & (*) & (*) \\ \Sigma_{\mathrm{ij}} & -\zeta & (*) \\ \Psi_{ij}^{12} & (0) & \Psi^{22} \end{pmatrix}$$

with

$$\Psi_{ij}^{12} = diag\begin{bmatrix} B_iK_j & P_{21} & & & & \\ P_1 & B_{i-j}K_j & P_{22} & 0 & 0 & 0 \\ B_iK_j & P_{21} & P_{22} & & & \\ CB_{i-j}K_j & CB_{i-j}K_j & & & & \end{bmatrix}$$

$$\Psi^{22} = -diag(I \;\; I \;\; I \;\; I \;\; I \;\; I\, I \;\; I \;\; I \;\; I\,)$$

$$\zeta = \mathrm{diag}\left[\lambda_1^a I \;\; \lambda_1^{\mathrm{b}} I \;\; (\lambda_1^b + I + \lambda_4^b + \lambda_5^b)I \;\; I \;\; \lambda_4^a I \;\; I \;\; (\lambda_3^b + \lambda_5^b)I \;\; \lambda_2^a I \;\; (\lambda_3^b)^{-1} I \;\; (\lambda_4^b)^{-1} I \;\; \lambda_5^b I \;\; \lambda_3^a I \;\; (\lambda_1^b)^{-1} I \; (\lambda_3^b)^{-1} I \;\; \lambda_6^b I \;\; (\lambda_5^b)^{-1} I\right]$$

## Fault tolerant tracking control strategy

with

$$\Sigma_{ij} = diag\begin{bmatrix} M_a^T P_1 & M_b^T P_{21} & 0 & M_a^T & M_b^T P_1 & 0 \\ M_b^T P_1 & M_b^T P_2 & M_b^T P_{21} & M_b^T & M_b^T P_{21} & 0 \\ N_{bi} K_j & N_{bi} K_j & M_b^T C^T P_2 & M_a^T C^T P_{22} & M_b^T C^T P_{22} & 0 \\ 0 & N_{ai} P_{21} & 0 & 0 & M_b^T & 0 \end{bmatrix}$$

$$\Psi_{ij}^{11} = \begin{pmatrix} I & * & * & * & * & * \\ 0 & \Im(P_{21}^T A_j) + \Im(W_1 C) + I & * & * & * & * \\ 0 & -P_{22} C A_j - W_2 C - W_1^T & \Im(W_2) + I & * & * & * \\ B_i K_j & -B_i K_j + A_{i-j}{}^{\boldsymbol{T}} P_{21} & -(C A_{i-j})^T P_{22} & \Im(A_i) & * & * \\ 0 & B_{i-j}{}^T P_{21} & -(C B_{i-j})^T P_{22} & B_{\boldsymbol{i}}^{\boldsymbol{T}} & -\eta^2 I & * \\ 0 & B_f^T P_{21} & -(C B_f)^T P_{22} & B_f^T & 0 & -\eta^2 I \end{pmatrix}$$

$$\Xi_{ij} = \begin{pmatrix} \lambda_1^a N_{ai}^T N_{ai} + \lambda_1^b N_{bi}^T N_{bi} & * & * & * & * & * \\ 0 & N_{bi}^T N_{bi} & * & * & * & * \\ 0 & 0 & \lambda_3^a N_{ai}^T N_{ai} + \lambda_6^b N_{bi}^T N_{bi} & * & * & * \\ 0 & 0 & 0 & \lambda_4^a N_{ai}^T N_{ai} + \lambda_2^a \mathrm{M}_a \mathrm{M}_a^T + \lambda_5^b N_{bi}^T N_{bi} & * & * \\ 0 & 0 & 0 & 0 & 0 & * \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

**The observer gains:**

$$R = (P_{21}^{-1} W_2 - C P_{22}^{-1} W_1)^{-1}$$

$$\Theta = P_{21}^{-1} W_1 R$$

$$F_j = \begin{bmatrix} A_j & 0 \\ -C & -I_p \end{bmatrix},\ L = \begin{bmatrix} 0 \\ I_p \end{bmatrix},\ E = \begin{bmatrix} I_n + \Theta C & \Theta \\ RC & R \end{bmatrix}$$

## Numerical example

# Numerical example

## Numerical example

Uncertain T-S faulty system

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2} \mu_i(\xi(t))\left(\mathbb{A}_i x(t) + \mathbb{B}_i u(t)\right) + B_d d(t) \\ y(t) = Cx(t) + D_f f(t) \end{cases}$$

with

$$A_1 = \begin{bmatrix} -2 & 1 \\ 2 & -3 \end{bmatrix},\ A_2 = \begin{bmatrix} -3 & 2 \\ 2 & -3 \end{bmatrix},\ B = \begin{bmatrix} 1 & 0.5 \\ 1 & 2 \end{bmatrix},\ B_d = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},\ D_f = \begin{bmatrix} 4 & 0 \\ 0 & 8 \end{bmatrix}$$

The uncertain matrices:

$$\Delta A_i = M_a \gamma(t) N_{ai} \quad \text{and} \quad \Delta B_i = M_b \gamma(t) N_{bi}$$

$$\text{with } i = 1{,}2\text{: } M_a = N_{ai} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},\ M_b = N_{bi} = [0.1 \quad 0].$$

The uncertainties are defined by:

$$\gamma(t) = sin(1.5t).$$

## Numerical example

Uncertain T-S faulty system

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2} \mu_i(\xi(t)) \left(\mathbb{A}_i x(t) + \mathbb{B}_i u(t)\right) + B_d d(t) \\ y(t) = Cx(t) + D_f f(t) \end{cases}$$

Membership functions:

$$\mu_1(x_1(t)) = \left(1 - tanh\left(0.5 - x_1(t)\right)\right)/2$$
$$\mu_2(x_1(t)) = 1 - \mu_1(x_1(t))$$

The fault signals

$$f_1(t) = 0.1sin10(t-6) \qquad occurs\ at\ 5sec \le t \le 9sec$$

$$f_2(t) = \begin{cases} 0.02(t-1) & 12sec \le t < 14sec \\ 0.01(t-1) & 14sec \le t \le 16sec \\ 0 & otherwise \end{cases}$$

## Numerical example

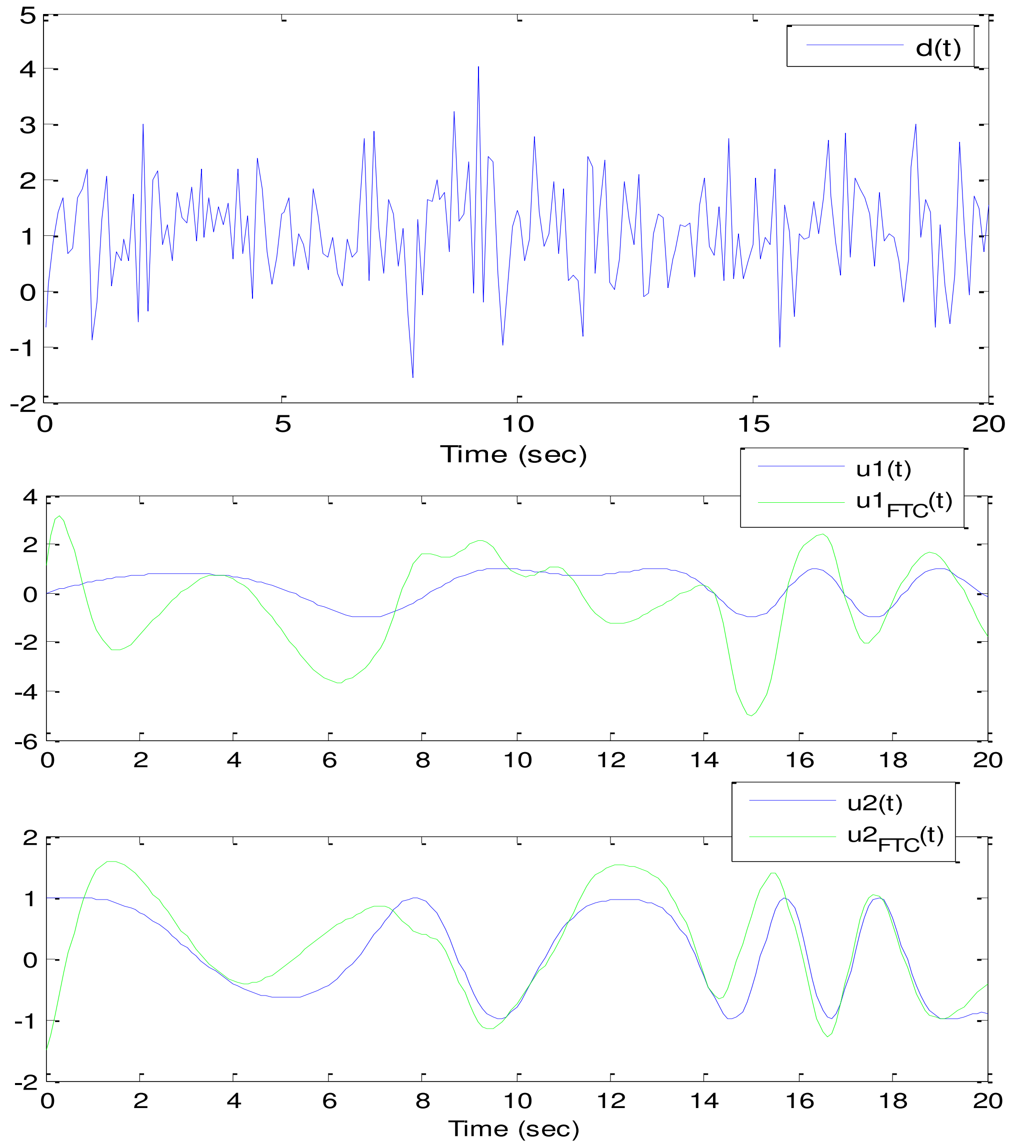



**Figure**. Disturbance (top), inputs (down)

## Numerical example

Controller and Observer gains

$$K_1 = \begin{bmatrix} 32.7680 & -1.3849 \\ -1.3849 & -7.8231 \end{bmatrix}; K_2 = \begin{bmatrix} -36.0529 & 4.0075 \\ 4.0075 & 5.9421 \end{bmatrix}$$

$$L = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^T$$

$$F_1 \begin{pmatrix} -2 & 1 & 0 & 0 \\ 2 & -3 & 0 & 0 \\ -0.1 & 0 & -1 & 0 \\ 0 & -0.1 & 0 & -1 \end{pmatrix}, F_2 \begin{pmatrix} -3 & 2 & 0 & 0 \\ 2 & -3 & 0 & 0 \\ -0.1 & 0 & -1 & 0 \\ 0 & -0.1 & 0 & -1 \end{pmatrix}$$

$$R = \begin{pmatrix} 0.1193 & 0.0090 \\ 0.0120 & 0.0614 \end{pmatrix}; \Theta = \begin{pmatrix} -9.9702 & -0.0563 \\ 0.0860 & -9.9357 \end{pmatrix}$$

$$E = \begin{pmatrix} 0.0030 & -0.0056 & -9.9702 & -0.0563 \\ 0.0086 & 0.0064 & 0.0860 & -9.9357 \\ 0.0119 & 0.0009 & 0.1193 & 0.0090 \\ 0.0012 & 0.0061 & 0.0120 & 0.0614 \end{pmatrix}$$

## Numerical example

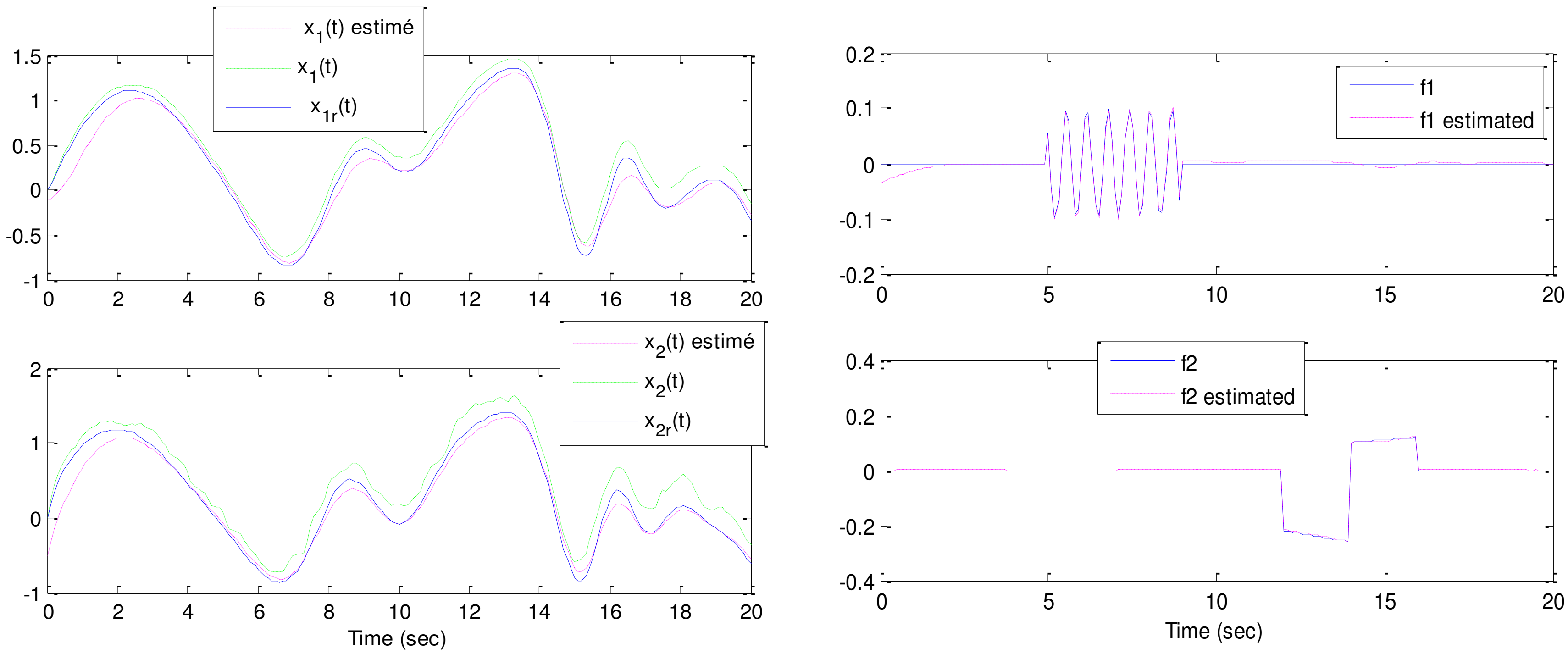



**Figure** . Faults ($f_1$, $f_2$) and their estimates (right), uncertain states ($x_1$, $x_2$) , reference states ($x_{1r}$, $x_{2r}$) and their estimates (left)

## Conclusions and perspectives

### Conclusions

- Model reference tracking control of faulty nonlinear systems represented by uncertain T-S structure has been considered
- A sensor fault tolerant control scheme based on a descriptor observer with a guaranteed $\mathcal{L}_2$ performance is proposed
- Convergence conditions expressed in an optimisation problem with LMI constraints

### Perspectives

- Extension of that work to nonlinear systems with time varying parameters
- Extension to the case of discrete time T-S models with uncertainties

Thanks for your attention