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Efficiency of the Flow in the Circular Pipe

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ABSTRACT

Design engineers are often faced with the complex task of designing new collection systems. Methods based on Manning's equation are frequently used due to the availability of tables and graphs which simplify the calculations. These methods lack accuracy except when laborious numerical methods are utilized. The design of a collection system seeks the computation of a diameter which produces an accepted velocity value without considering the water level in the computed pipe. Yet, the flow efficiency whether volumetric or circulation is an important design criteria. By considering the latter an increase in the volumetric capacity and circulation capacity of the flow in the pipe can be obtained. In this research, a new concept for the design of partially full pipe is proposed. It is based on Manning's equation and produces more efficient flow in pipe, i.e., the pipe is as fully exploited as possible.

Key words: Pipes, steady uniform flow, pipe efficiency, Manning equation, pipe efficiency

INTRODUCTION

Sewerage can be defined as the evacuation of waste water rapidly and far away from populated areas and business districts without stagnation in pipes. The best design of sewer evacuation systems starts by studying the parameters which effect their operations, including technical, environmental and economical ones (McGhee and Steel, 1991).

The flow in the collection system is usually considered uniform and steady. This type of flow has been investigated extensively by several researchers, where a number of approaches have been proposed including graphical methods (Camp, 1946; Chow, 1959; Swarna and Modak, 1990), semi-graphical solutions (Zeghadnia *et al.*, 2009) and nomograms (McGhee and Steel, 1991) or tables (Chow, 1959). However, such approaches are usually considered limited and most of them are applicable only to limited conditions. Numerical solutions are usually preferred in practice but these are difficult to apply and need to go through relatively lengthy trial and errors procedures.

A number of researchers have attempted to propose explicit equations for the computation of normal depth (Barr and Das, 1986; Saatci, 1990; Swamee and Rathie, 2004; Achour and Bedjaoui, 2006). Other authors prefer to simulate pressurized flow as free surface flow using the Preissmann Slot Method, hence, they can model the transition from free surface flow to surcharged state and vice versa (Cunge *et al.*, 1980; Garcia-Navarro *et al.*, 1994; Capart *et al.*, 1997; Ji, 1998; Trajkovic *et al.*, 1999; Ferreri *et al.*, 2010).

The majority of research in this area is heavily focused on the determination of flow parameters, without looking at the performance of the flow inside the pipe. The concept of efficient pipe has not previously been explicitly discussed. The authors think that this is the first time this idea has been used in the direct calculation of pipes which should draw the interest of researchers and designers alike. The efficiency of flow, therefore the efficiency of pipe is introduced as a measurable characteristic. Accordingly, the pipe will flow with maximum use of water surface, i.e., fully exploiting its surface area while respecting the technical requirements, especially in terms of velocity.

In this study we will shed some light on certain important technical considerations regarding the determination of hydraulic and geometrical parameters of partially filled pipes. The analysis takes into account other parameters like the slope, diameter, velocity and pipe flow efficiency using explicit solutions. Also, the limitations of the proposed solutions will be discussed.

MANNING EQUATION

Circular pipes are widely used for sanitary sewage and storm water collection systems. The design of sewer networks is generally based on the Manning model (Manning, 1891), where the flow section is mostly partially filled. The manning formula is commonly used in practice and is assumed to produce the best results when properly applied (Saatci, 1990; Zeghadnia *et al.*, 2014a, b). The usage of Manning model assumes the flow to be steady and uniform, where the slope, cross-sectional flow area and velocity are not related to time and are constant along the length of the pipe being analyzed (Carlier, 1980). The Manning formula (Manning, 1891) used to model free surface flow can be written as follow:

$$Q = \frac{1}{n} R_h^{2/3} A S^{1/2} \qquad (1)$$

or

$$V = \frac{1}{n} R_h^{2/3} S^{1/2} \qquad (2)$$

Where:

Q : Flow rate (m^3 sec^{-1})
R_h : Hydraulic radius (m)
n : Pipe roughness coefficient (Manning n) (sec $m^{-1/3}$)
A : Cross sectional flow area (m^2)
S : Slope of pipe bottom, dimensionless
V : Flow velocity (m sec^{-1})

Equation 1 and 2 can be written as functions of water surface angle shown in Fig. 1 as follow: From Fig. 1:

$$Q = \frac{1}{n}\left(\frac{D^8}{2^{13}}\right)^{1/3}\left[\frac{(\theta - \sin\theta)^5}{\theta^2}\right]^{1/3} S^{1/2} \qquad (3)$$

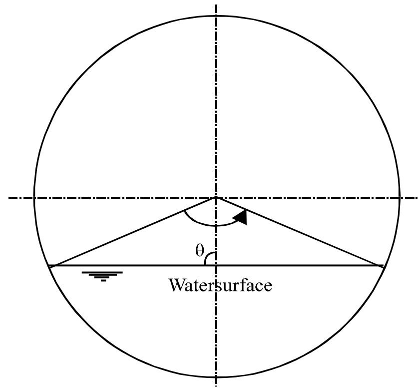

Fig. 1: Water surface angle

$$V = \frac{1}{n}\left(\frac{D}{4}\right)^{2/3}\left[\frac{(\theta - \sin\theta)}{\theta}\right]^{2/3} S^{1/2} \tag{4}$$

$$A = \frac{D^2}{8}(\theta - \sin(\theta)) \tag{5}$$

$$P = \theta\frac{D}{2} \tag{6}$$

$$R_h = \frac{A}{P} = \frac{D}{4}\left(1 - \frac{\sin(\theta)}{\theta}\right) \tag{7}$$

Where:

D : Pipe diameter (m)

r : Pipe radius:

$r = \frac{D}{2}$ (m)

P : Wetted perimeter (m)

θ : Water surface angle (Radian)

Equation 3 and 4 for known values of flow Q, roughness n, slope S and diameter D can be solved only after a series of long iterations (Giroud *et al*., 2000). Equation 4 can be substituted by Eq. 8 (Zeghadnia *et al*., 2009):

$$V = a\theta^{-2/5} \tag{8}$$

Where:

$$a = \frac{1}{n}\left(\frac{D}{4}\right)^{2/3} K^{2/3} S^{1/2}$$

$$K = \left[\left(\frac{nQ}{S^{1/2}}\right)^3 \left(\frac{2^{13}}{D^8}\right)\right]^{1/5}$$

Therefore:

$$V = \left(\left(\frac{S^{1/2}}{n}\right)^3 \left(\frac{2Q}{D}\right)^2\right)^{1/5} \theta^{-2/5} \tag{9}$$

Equation 5 and 7 take the new forms as follows:

$$A = \left(\frac{D}{2}\right)^{2/5} \left(\frac{nQ}{S^{1/2}}\right)^{3/5} \theta^{2/5} \tag{10}$$

$$R_h = \left(\frac{2nQ}{D\, S^{1/2}}\right)^{3/5} \theta^{-3/5} \tag{11}$$

METHODOLOGY

Estimation of volumetric or circulation efficiency: In order to simplify the computation, the calculation of pipe diameter is done frequently with the assumption that the pipe is flowing just full (under atmospheric pressure). Either flow or flow velocity can have maximum values which correspond to certain water level in the pipe (Camp, 1946). Below or above this level, the flow or the velocity values decrease which means that the pipe is not flowing with its maximum efficiency. For best hydraulic design of sanitary sewage and storm water collection systems, it is not enough to determine the diameter which produces an acceptable flow velocity, but it is also necessary to determine the best diameter which allows higher efficiency and ensure that the pipe is fully exploited. To estimate the volumetric efficiency in pipe, we propose the flowing equation:

$$Q_{ef}\,(\%) = \left(1 - \frac{Abs(Q_{max} - q_r)}{Q_{max}}\right) \times 100 \tag{12}$$

Where:

Q_{ef} : Volumetric efficiency (%)
Q_{max}: Maximum flow ($m^3\ sec^{-1}$)
q_r : Flow in pipe ($m^3\ sec^{-1}$)

And to compute the circulation efficiency in pipe, we propose the flowing formula:

$$V_{ef}\,(\%) = \left(1 - \frac{Abs(V_{max} - V_r)}{V_{max}}\right) \times 100 \tag{13}$$

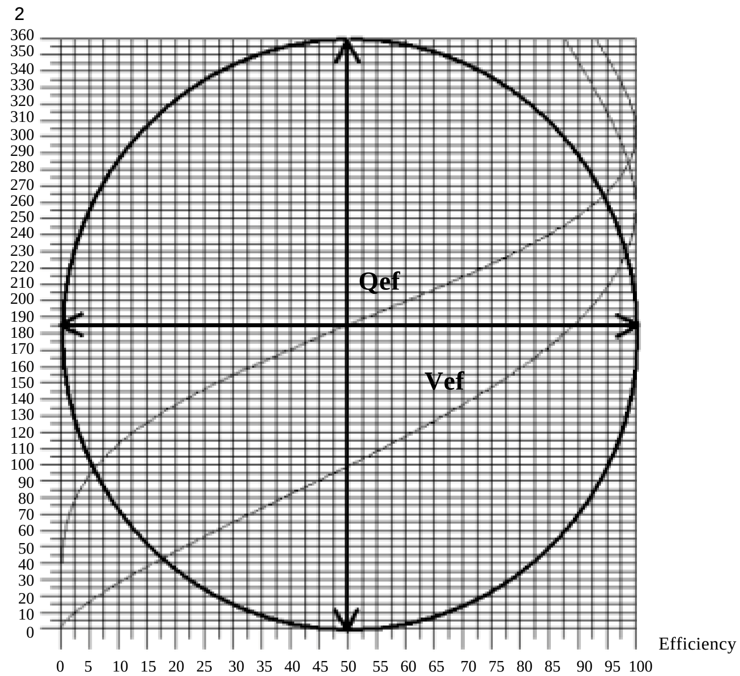

Fig. 2: Volumetric and circulation efficiency in circular pipe

Where:

V_{ef} : Circulation efficiency (%)

V_{max} : Maximum velocity (m^2 sec^{-1})

V_r : Velocity in pipe (m^2 sec^{-1})

The volumetric and circulation efficiencies can be better explained using the graphical representation shown in the Fig. 2.

Figure 2 shows that the volumetric or circulation efficiency depends on the level of filling of the pipe and they do not vary in the same manner.

For $0° \leq \theta \leq 40°$, the volumetric efficiency is practically zero while for $40° \leq \theta \leq 180°$, it is less than 50%. For $\theta = 185°$, the efficiency equals 50% and it reaches its maximum value, $Q_{ef} \cong 100\%$, at $\theta = 308°$. For $308° \leq \theta \leq 360°$ the volumetric efficiency decreases to reach a value of 93.09%.

On the other hand the variation of the circulation efficiency is more rapid than the volumetric efficiency. For $0° \leq \theta \leq 40°$ the circulation efficiency can reach 20% and for $40° \leq \theta \leq 180°$ the efficiency reaches 85%. The circulation efficiency reaches its maximum value, $V_{ef} \cong 100\%$, at $\theta = 257°$. For $257° \leq \theta \leq 360°$ the circulation efficiency decreases to reach a value of 87.74%. Table 1 presents more details on the variation of both efficiencies as functions of θ.

Example: In this example we calculate the volumetric and circulation efficiencies for pipes with velocity $V_r = 0.88$ m sec^{-1}, $q_r = 0.15$ m^3 sec^{-1}, in a 500 mm pipe diameter, $Q_{max} = 0.256$ m^3 sec^{-1}, $Q_{full} = 0.238$ m^3 sec^{-1}, $V_{full} = 1.212$ m sec^{-1}, $V_{max} = 1.30$ m sec^{-1}.

Table 1: Volumetric and the circulation efficiency as function of water surface angle

Water surface angle θ	Volumetric efficiency (Q_{ef})	Circulation efficiency (V_{ef})
0	0.0000	0.0000
1	0.0000	0.1202
2	0.0000	0.3028
3	0.0000	0.5199
4	0.0000	0.7630
5	0.0000	1.0273
10	0.0004	2.5867
15	0.0022	4.4360
20	0.0077	6.4983
25	0.0202	8.7302
30	0.0441	11.1016
45	0.2487	18.8458
60	0.8313	27.2175
90	4.2964	44.6410
120	12.7243	61.4284
150	27.1924	76.1254
240	84.7992	99.4167
257.584	92.3528	100. 000
260	93.1948	99.9895
295	99.9107	97.7469
308.3236	100.0000	96.0693
360	93.0919	87.7467

Using Eq. 12 and 13, we find that $Q_{ef} = 58.59$ and $V_{ef} = 67.68\%$. Hence, this pipe is not efficient enough both in terms of volume and circulation. In this example, although the velocity is technically acceptable, this pipe is not flowing efficiently. Hence we need to find a better solution to insure high efficiency of the pipe which will be discussed in the following sections.

RESULTS AND DISCUSSION

Maximum volumetric efficiency: The efficiency is discussed in the following paragraphs in terms of pipe volume occupancy. The higher the latter, the more efficient the pipe is.

Maximum flow condition: When cross sectional flow area A increases, it reaches its maximum value "A_{max}" with maximum volumetric efficiency at θ = 308.3236 (Zeghadnia *et al.*, 2009). From Eq. 3:

$$Q_{max} = 0.3349288\frac{D^{8/3}S^{1/2}}{n} \tag{14}$$

For a pipe flowing full, the flow "Q" is expressed as follow:

$$Q_p = 0.3117909\frac{D^{8/3}S^{1/2}}{n} \tag{15}$$

When we combine Eq. 14 and 15 we obtain the following:

$$Q_{max} = 1.067795120Q_P \tag{16}$$

Equation 16 presents the relationship between the flow for filled pipe and the maximum flow which, for any section is possible only if the following condition is achieved (Carlier, 1980):

$$3PdA\text{-}AdP = 0 \tag{17}$$

where, (P is the wetted perimeter):

$$P = \theta \Rightarrow dp = rd\theta \tag{18}$$

$$A = \frac{r^2}{2}(\theta - \sin\theta) \Rightarrow dA = \frac{r^2}{2}(1 - \cos\theta)d\theta \tag{19}$$

If we substitute the wetted perimeter "P", cross sectional flow area "A" and their derivatives in Eq. 17, we obtain the following:

$$3\frac{dA}{A} = \frac{dp}{P} \Rightarrow A^3 = P \tag{20}$$

If we combine Eq. 7 and 20, then Eq. 1 becomes:

$$Q = \frac{S^{1/2}}{n}\frac{A^{5/3}}{P^{2/3}} = \frac{S^{1/2}}{n}p^{-1/9} \tag{21}$$

From Eq. 21, the wetted perimeter can be rewritten as follow:

$$P = \left(\frac{S^{1/2}}{nQ}\right)^9 \tag{22}$$

By combining Eq. 6 and 22 we obtain the following:

$$D = \frac{2}{\theta_{Qmax}}\left(\frac{S^{1/2}}{nQ}\right)^9 \tag{23}$$

Equation 23 can also be rewritten as follow:

$$D = 0.372\left(\frac{S^{1/2}}{nQ}\right)^9 \tag{24}$$

The use of Eq. 24 to compute the diameter, for flow maximum is simple and direct when the roughness n and the slope S are known.

In the case where the slope S is unknown, Eq. 25 gives an explicit solution, if the flow Q, roughness n and diameter D are known.

$$S = \left(nQ \left(\frac{D}{0.372} \right)^{1/9} \right)^2 \tag{25}$$

Flow velocity limits: By combining Eq. 2, 7 and 20 we obtain:

$$V = \frac{S^{1/2}}{n} P^{-4/9} \tag{26}$$

If we substitute the wetted perimeter expression given in Eq. 22, into Eq. 26, we obtain the following:

$$V = \frac{S^{1/2}}{n} \left(\left(\frac{S^{1/2}}{nQ} \right)^9 \right)^{-4/9} = \left(\frac{n}{S^{1/2}} \right)^2 Q^4 \tag{27}$$

The combination between Eq. 24 and 27 produces:

$$V = \frac{S^{1/2}}{n} \left(\frac{0.372}{D} \right)^{4/9} \tag{28}$$

From Eq. 27, the cross sectional area A can be rewritten as follow:

$$A = Q/V = \left(\frac{S^{1/2}}{n} \right)^3 Q^{-3} = R_R^3 Q^{-3} \tag{29}$$

We call " R_R " the resistance rate which can be computed using Eq. 27 or 28 for maximum and minimum values of the flow velocity, respectively. Equation 27 and 28 are applied only for the range of values given in Table 2 and 3 in which the flow velocity varies between 0.5 m $sec^{-1} \leq V \leq$ 5 m sec^{-1} (Satin and Selmi, 2006). In practice, the pipe diameters ranges generally between: 10 mm$\leq D \leq$ 2100 mm.

Table 2 and 3 presents the solutions for Eq. 27 and 28. By comparing the flow velocities in Table 2 and 3 we can conclude that the resistance rate R_R influences remarkably these values. For diameters that vary in range between 10 mm$\leq D \leq$ 250 mm, the minimal value of R_R should not be lower than 0.4. This yields a variation in the flow in the range given by the following relationship:

Table 2: Flow velocity limits as a function of diameter and flow for the minimum value of R_R = 0.4 and 10 mm≤D≤ 250 mm

D (mm)	Q (m^3 sec^{-1})	V(Q) (m sec^{-1})	V(D) (m sec^{-1})
10	0.60	2.00	2.00
12	0.59	1.84	1.84
16	0.57	1.62	1.62
20	0.55	1.47	1.47
25	0.54	1.33	1.33
32	0.53	1.19	1.19
40	0.51	1.08	1.08
50	0.50	0.98	0.98
63	0.49	0.88	0.88
75	0.48	0.81	0.82
90	0.47	0.75	0.75
100	0.46	0.72	0.72
110	0.46	0.69	0.69
125	0.45	0.65	0.65
140	0.45	0.62	0.62
160	0.44	0.58	0.58
200	0.43	0.53	0.53
225	0.42	0.50	0.50
250	0.42	0.50	0.50

Table 3: Flow velocity limits as a function of diameter and flow for the maximum value of R_R =1 and 10 mm≤D≤ 250 mm

D (mm)	Q (m^3 sec^{-1})	V(Q) (m sec^{-1})	V(D) (m sec^{-1})
10	1.49	4.99	4.99
12	1.46	4.60	4.60
16	1.42	4.05	4.05
20	1.38	3.67	3.67
25	1.35	3.32	3.32
32	1.31	2.97	2.98
40	1.28	2.69	2.69
50	1.25	2.44	2.44
63	1.22	2.20	2.20
75	1.19	2.04	2.04
90	1.17	1.88	1.88
100	1.16	1.79	1.79
110	1.14	1.72	1.72
125	1.13	1.62	1.62
140	1.11	1.54	1.54
160	1.10	1.45	1.45
200	1.07	1.32	1.32
225	1.06	1.25	1.25
250	1.05	1.19	1.19

$$0.42\ m^3\ sec^{-1} \le Q \le 0.6\ m^3\ sec^{-1}$$

The same diameter range accepts another boundary as maximum flow value for R_R =1. This generates the following flow values range:

$$1.05\ m^3\ sec^{-1} \le Q \le 1.49\ m^3\ sec^{-1}$$

Table 4: Flow velocity limits as function of diameter and flow for minimum $R_R(min) = 1.05$, 315 mm≤D≤ 2100 mm

D (mm)	Q ($m^3 sec^{-1}$)	V(Q) ($m sec^{-1}$)	V(D) ($m sec^{-1}$)
315	1.07	1.13	1.13
400	1.04	1.02	1.02
500	1.02	0.92	0.92
600	1.00	0.85	0.85
700	0.98	0.79	0.79
800	0.96	0.75	0.75
900	0.95	0.71	0.71
1000	0.94	0.68	0.68
1100	0.93	0.65	0.65
1200	0.92	0.62	0.62
1300	0.91	0.60	0.60
1400	0.91	0.58	0.58
1500	0.90	0.56	0.57
1600	0.89	0.55	0.55
1700	0.89	0.53	0.53
1800	0.88	0.52	0.52
1900	0.88	0.51	0.51
2000	0.87	0.50	0.50
2100	0.87	0.50	0.50

If we expand the range of variation in diameter: 315 mm≤D≤ 2100 mm while we keep the condition of flow velocity as indicated above, we obtain the following results given in Table 4 and 5. The latter present the variation of flow values as a function of the diameter and the limit values of R_R. We can summarize the variation of flow according to the variation of R_R as follow:

- For the minimum value of $R_R = 1.05$, the flow varies, according to Table 4 results as follow:

$$0.87 \text{ m}^3 \text{ sec}^{-1} \leq Q \leq 1.07 \text{ m}^3 \text{ sec}^{-1}$$

For the maximum value of $R_R = 4.64$, the flow varies, according to Table 5 results as follow:

$$3.83 \text{ m}^3 \text{ sec}^{-1} \leq Q \leq 4.73 \text{ m}^3 \text{ sec}^{-1}$$

Other results could easily be obtained using different values of R_R within its accepted limits.

Maximum circulation efficiency: In this section the efficiency of the pipe is treated based on the circulation of flow. We look at the variation of the circulation efficiency from different levels. Then we will present how to obtain the maximum exploitation of the pipe.

Condition of maximum Flow velocity: Flow under condition of maximum flow velocity is an important in sewage network drainage. In these types of flow condition it is imperative to check the following condition (Carlier, 1980):

$$PdA\text{-}Adp = 0 \tag{30}$$

Table 5: Flow velocity limits as function of diameter and flow for maximum R_R (max) = 4.64. 315 mm≤D≤2100 mm

D (mm)	Q (m^3 sec^{-1})	V(Q) (m sec^{-1})	V(D) (m sec^{-1})
315	4.73	5.00	5.00
400	4.60	4.49	4.49
500	4.49	4.07	4.07
600	4.40	3.75	3.75
700	4.33	3.50	3.50
800	4.26	3.30	3.30
900	4.21	3.13	3.13
1000	4.16	2.99	2.99
1100	4.11	2.87	2.87
1200	4.07	2.76	2.76
1300	4.04	2.66	2.66
1400	4.00	2.57	2.57
1500	3.97	2.50	2.50
1600	3.95	2.43	2.43
1700	3.92	2.36	2.36
1800	3.89	2.30	2.30
1900	3.87	2.25	2.25
2000	3.85	2.20	2.20
2100	3.83	2.15	2.15

Where:
P : Wetted perimeter (m)
A : Cross sectional flow area (m^2)

The combination between the Eq. 18, 19 and 30 gives the following:

$$-\theta\cos\theta+\sin\theta = 0 \tag{31}$$

Equation 31 can be solved iteratively. The use of the Bisection Method (Andre, 1995) gives the following results (where the absolute error equal to 10^{-6}): θ = 257, 584:

$$\frac{dA}{A} = \frac{dP}{P} \tag{32}$$

From Eq. 6, 10 and 32 and after many simplifications we obtain the following equation:

$$D = \frac{0.445\ nQ}{S^{1/2}} \tag{33}$$

Therefore, Eq. 10 can be rewritten as follow:

$$A = \frac{nQ}{S^{1/2}} \tag{34}$$

Table 6: Recommended limits of flow velocity as a function of diameter and flow for: R_R (min) = 0.5 and 10 mm≤D≤2100 mm

D (mm)	Q (m^3 sec^{-1})	V (m sec^{-1})	D (mm)	Q (m^3 sec^{-1})	V (m sec^{-1})
10	0.01	0.50	315	0.35	0.50
12	0.01	0.50	400	0.45	0.50
16	0.02	0.50	500	0.56	0.50
20	0.02	0.50	600	0.67	0.50
25	0.03	0.50	700	0.79	0.50
32	0.04	0.50	800	0.90	0.50
40	0.04	0.50	900	1.01	0.50
50	0.06	0.50	1000	1.12	0.50
63	0.07	0.50	1100	1.24	0.50
75	0.08	0.50	1200	1.35	0.50
90	0.10	0.50	1300	1.46	0.50
100	0.11	0.50	1400	1.57	0.50
110	0.12	0.50	1500	1.69	0.50
125	0.14	0.50	1600	1.80	0.50
140	0.16	0.50	1700	1.91	0.50
160	0.18	0.50	1800	2.02	0.50
200	0.22	0.50	1900	2.13	0.50
225	0.25	0.50	2000	2.25	0.50
250	0.28	0.50	2100	2.36	0.50

Equation 33 for known flow Q, roughness n and slope S, gives explicit solution for the diameter. The slope S can be also calculated directly by Eq. 35 if the flow Q, roughness n and diameter D are known parameters:

$$S = \left(\frac{2\ nQ}{4.49\ D}\right)^2 \tag{35}$$

According to Eq. 34, it is easy to deduce that the flow velocity is equal to the ratio of square root of the slope and roughness as follow:

$$V = \frac{S^{1/2}}{n} = \frac{0.445\ Q}{D} \tag{36}$$

From Eq. 36 and at first glance we can conclude that the flow velocity depends only on the slope and roughness. This is true in this case. However, this conclusion must be related to another reality, that this formula is conditioned by the fullness degree in the pipe which means the diameter used in Eq. 36 should be computed using Eq. 33 firstly.

Recommended limits: The proposed model of flow under condition of maximum velocity is governed by flow velocity limits which produce a succession of limits of the other parameters: Flow, slope and pipe roughness for the range of values presented in Table 6 and 7:

Table 7: Recommended limits of flow velocity as a function of diameter and flow for: R_R (max) = 5 and 10 mm≤D≤2100 mm

D (mm)	Q (m^3 sec^{-1})	V (m sec^{-1})	D (mm)	Q (m^3 sec^{-1})	V (m sec^{-1})
10	0.11	5.00	315	3.54	5.00
12	0.13	5.00	400	4.49	5.00
16	0.18	5.00	500	5.62	5.00
20	0.22	5.00	600	6.74	5.00
25	0.28	5.00	700	7.87	5.00
32	0.36	5.00	800	8.99	5.00
40	0.45	5.00	900	10.11	5.00
50	0.56	5.00	1000	11.24	5.00
63	0.71	5.00	1100	12.36	5.00
75	0.84	5.00	1200	13.48	5.00
90	1.01	5.00	1300	14.61	5.00
100	1.12	5.00	1400	15.73	5.00
110	1.24	5.00	1500	16.85	5.00
125	1.40	5.00	1600	17.98	5.00
140	1.57	5.00	1700	19.10	5.00
160	1.80	5.00	1800	20.22	5.00
200	2.25	5.00	1900	21.35	5.00
225	2.53	5.00	2000	22.47	5.00
250	2.81	5.00	2100	23.60	5.00

From the parameters values shown in Table 6 and 7, we can easily conclude that the resistance rate R_R is an important parameter, where it can allow for the enlargement or the narrowing of the range of validity. In the case of maximum velocity the equations of applicability can be presented as follow:

- For minimal value of R_R = 0.5 and for diameters range of 10 mm≤D≤ 2100 mm, the flow varies as follow:

$$0.01\ m^3\ sec^{-1} \leq Q \leq 2.36\ m^3\ sec^{-1}$$

- If R_R = 5 and 10 mm ≤D≤ 2100 mm, the flow varies as follow:

$$0.11\ m^3\ sec^{-1} \leq Q \leq 23.60\ m^3\ sec^{-1}$$

From the above and in a similar way to the case of flow under condition of maximum velocity or maximum flow, it's imperative to respect the variation of the resistance rate R_R which gives afterwards acceptable values for flow velocity and not necessary desired flow, because each range of R_R generates different range of flow. The range of flow values are given as follows:

- Case of flow max:

$$Q_D = 250\ mm \leq Q_{known} \leq Q_D = 10\ mm$$

Or:

$$Q_D = 2100 \text{ mm} \leq Q_{known} \leq Q_D = 315 \text{ mm}$$

- Case of velocity max:

$$Q_D = 10 \text{ mm} \leq Q_{known} \leq Q_D = 2100 \text{ mm}$$

Let us take practical field scenarios through the following two examples.

Example 1: A pipe with manning coefficient n = 0.013, slope S = 0.02%, transport a flow of 1.05 m^3 sec^{-1}. Compute the pipe diameter for maximum volumetric efficiency.

Solution: First we must check if the value of the resistance rate R_R is respected so we can use the model:

$$1.05 \leq R_R = \frac{S^{1/2}}{n} = 1.08 \leq 4.64$$

The resistance rate belongs to the allowable range. From Table 3 and 4, we can conclude that diameter varies as follows:

$$315 \text{ mm} \leq D \leq 2100 \text{ mm}$$

Checking the flow range: From Eq. 24 it is easy to compute Q_D = 315 mm and Q_D = 2100 mm.

$$Q_D = 315 \text{ mm} = \left(\frac{0.372}{D}\right)^{1/9} \frac{i^{1/2}}{n} = 1.10 \text{ m}^3 \text{ sec}^{-1}$$

$$Q_D = 2100 \text{ mm} = \left(\frac{0.372}{D}\right)^{1/9} \frac{i^{1/2}}{n} = 0.89 \text{ m}^3 \text{ sec}^{-1}$$

$$0.89 \text{ m}^3 \text{ sec}^{-1} \leq Q = 1.05 \leq 1.10 \text{ m}^3 \text{ sec}^{-1}$$

Q belongs to the allowable range.
From Eq. 24 the diameter is calculated as:

$$D = 0.372\left(\frac{i^{1/2}}{nQ}\right)^9 \cong 500 \text{ mm}$$

Checking of the flow velocity: From Eq. 27 we obtain the following:

$$V = 0.95 \text{ m sec}^{-1}$$

The flow velocity value is acceptable, the same for the diameter which will produce, with the other parameters, the maximum flow (Which corresponded to fullness degree Q_{max}).

Example 2: Let us to use the same data for the previous example to calculate the new diameter in case of maximum efficiency of flow circulation in pipe.

Solution: Checking for allowable R_R range:

$$0.5 \leq \frac{S^{1/2}}{n} = 1.08 \leq 5$$

Therefore, the diameter varies as follow:

$$10\text{ mm} \leq D \leq 2100\text{ mm}$$

Checking for the flow range: Eq. 33 allows the calculation of Q_D = 10 mm and Q_D = 2100 mm.

$$Q_D = 10\text{ mm} = \left(\frac{D}{0.445}\right)\frac{i^{1/2}}{n} = 0.02\text{ m}^3\text{ sec}^{-1}$$

$$Q_D = 2100\text{ mm} = \left(\frac{D}{0.445}\right)^{1/9}\frac{i^{1/2}}{n} = 5.10\text{ m}^3\text{ sec}^{-1}$$

$$0.02\text{ m}^3\text{ sec}^{-1} \leq Q = 1.05 \leq 5.10\text{ m}^3\text{ sec}^{-1}$$

Hence, the flow is within the allowable range.

Computation of the pipe diameter from Eq. 33 the pipe diameter equals to:

$$D = \frac{0.445\text{ nQ}}{i^{1/2}} \cong 400\text{mm}$$

From the above, the pipe diameter D is a known parameter, the flow velocity depends only on the slope S and roughness n and from Eq. 36 we obtain the following:

$$V = \frac{i^{1/2}}{n} = 1.08\text{ m sec}^{-1}$$

The flow velocity is within the acceptable range.

CONCLUSION

A new conception of the design of partially full flow in circular pipe is proposed using the new concept of volumetric and circulation efficiency. Two types of flow are considered: Flow under condition of maximum flow and flow under maximum velocity respectively. These are important criteria for the waste water evacuation. For both cases, direct and easy solutions have been elaborated to calculate the pipe diameter, flow velocity and slope. In the first the diameter and slope can be calculate with Eq. 24 and 25. For the second case Eq. 33 and 35 are recommended. For each case the computation of flow velocity is possible.

The limitation of the solution range has been discussed too. The proposed equations are elaborated to obtain high efficiency of flow in circular pipes while meeting the technical requirements.

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NOTATION

Q : Flow rate in m^3 sec^{-1}
R_h : Hydraulic radius
n : Pipe roughness coefficient (Manning n)
A : Cross sectional flow area
S : Slope of pipe bottom, dimensionless
V : Flow velocity m sec^{-1}
r : Pipe radius, let's: $r = D/2$
D : Pipe diameter
P : Wetted perimeter
θ : Water surface angle
Q_{ef} : Volumetric efficiency
Q_{max} : Flow max
q_r : Flow in pipe
V_{ef} : Circulation efficiency
V_{max} : Velocity max
A_{max} : Velocity in pipe
A_{max} : Cross sectional area correspond to Q_{max}
Q_p : Flow in full section
θ_{Qmax} : Water surface angle correspond to Q_{max}
R_R : Resistance rate

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