# Design of Collection System Parameters Using Known Reference Pipe Method (KRPM) 

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#### Abstract

The storm water drainage network is generally calculated based on the Manning equation, where the slope, roughness of the pipe wall, and flow are known, while conversely the velocity, diameter, and hydraulic radius are unknown characteristics, although they are very important for the work done by a hydraulic engineer who needs these parameters to find their values, including the students taking coursework relating to waste-water engineering. The computation of these parameters in partially full pipes and based on the Manning equation is implicit and needs to be computed using iterative and laborious methods. In this paper, a new, simple and easy method is presented based on a reference pipe with known characteristics (Known Reference Pipe Method: KRPM), as well as the effect of the up-pipe parameters on the down-pipes according to each case that is possible through the watershed drainage system arrangement, for both full and partially filled circular pipes.


Key words: Storm water drainage network, Manning equation, uniform and steady flow, watersheds arrangement, KRPM.

## Introduction

The storm sewer system is responsible for evacuating urban run-off. Most such systems are built with circular pipes (McGhee, 1991; Zeghadnia, 2015). The need to determine its characteristics is frequently encountered in the work of hydraulic engineers, undergraduate and graduate students. Generally, the flow is assumed as steady and uniform, which allows the use of the Manning equation (Manning, 1891), which is considered the preferred model to describe flow in storm sewer systems. The storm water sewer is designed to
capture and transport all rain events that are equal to or smaller than the design storm. In most of these designs, the pipe dimensions are designed for the maximum flow expected. Several authors prefer storm water drainage systems designed to convey rain events in free surface flow conditions, because this implies that the flow is transported by the action of gravity (Bourrier, 1997; Marc and Bechir, 2006; McGhee, 1991; Zeghadnia et al., 2015).

In general a circular cross-section is considered in the design of the storm water network. However, other shapes have been documented by a number of

[^0]authors (Donkin, 1937; French, 1915; Kuhn, 1976; Schmidt, 1976; Thormann, 1941, 1944) as reported by Willi (2010). The computation of storm sewer network parameters is scattered in different papers; several have proposed explicit solutions for the computation in a pressurised pipe (Bdjaoui et al., 2010; Christos et al., 2013; Cléberet al., 2019; Rajaratnam, 1960; Swamee et al., 2007; Wang et al., 2012; Wiggert, 1972; Zeghadnia, 2007), while in the case of the surface flow type many papers have been published (Akgray, 2005; Achour, 2006; Barr and Das, 1986; Boukhari et al., 2020; Elhakeem et al., 2017; Fukuchi, 2006; Giroud et al., 2000; Swarna et al., 1990; Saâtçi, 1990; Swamee et al., 2001; Tommy 2007; Zeghadnia, 2014b, 2014c; Zeghadnia et al., 2009, 2015, 2017). Ashok et al. (2008) proposed a direct method for the determination of the hydraulic and geometric elements of partly full flowing circular pipes based on the known parameters of the pipes, such as the filling rate of the pipe and the diameter, but no error was calculated and it is sometimes not possible to know these data in advance, so this can be qualified as a classical method rather than a new approach.

Through this paper, based on the Manning model, a new approach is proposed for computing the storm sewer network parameters in pipes arranged in parallel and series form. The Known Reference Pipe Method (KRPM) is much simpler and more accurate than the other methods, for the whole range of surface water angles, $0^{\circ} \leq \theta \leq 360^{\circ}$, where the computed pipes have the same filling rate as the reference pipes.

## The New Approach

Urban watersheds can be disposed in series form or parallel form, which influences the conception of the storm sewer system. In series form cases, the watersheds are arranged as shown in Figure 1.


Figure 1: Watershed arranged in series form.
where Pipe fl-f2: responsible for drainage watershed $\mathrm{N}^{\circ} 01$, to be called pipe 1; Pipe $\mathrm{f} 2-\mathrm{f} 3$ : responsible for drainage watershed $\mathrm{N}^{\circ} 01$ and $\mathrm{N}^{\circ} 02$, called pipe 3; $\mathrm{SS}_{1}$ : surface of watershed $\mathrm{N}^{\circ} 01 ; \mathrm{SS}_{2}$ : surface of watershed $\mathrm{N}^{\circ} 02$ and $\mathrm{SS}_{3}$ : surface of equivalent watershed.

The relation between $\mathrm{SS}_{1}, \mathrm{SS}_{2}$ and $\mathrm{SS}_{3}$ can be described by Equation (1):

$$
\begin{equation*}
\mathrm{SS}_{3}=\mathrm{SS}_{2}+\mathrm{SS}_{1} \tag{1}
\end{equation*}
$$

Thus, in the case of parallel arrangement of the watersheds, the pipes of the storm sewer network are disposed as shown in Figure 2.


Figure 2: Watershed arranged in parallel form.
where M1N1: pipe responsible for evacuating watershed $\mathrm{N}^{\circ} 01$ (represented by surface CC 1 ), referred to as pipe 1. M2N1: pipe responsible for evacuating watershed $\mathrm{N}^{\circ} 02$ (represented by surface CC2), called pipe 2. N1N2: pipe responsible for evacuating the equivalent watershed (represented by surface CC3), called pipe 3.
where:

$$
\begin{equation*}
\mathrm{CC}_{3}=\mathrm{CC}_{1}+\mathrm{CC}_{2} \tag{2}
\end{equation*}
$$

The characteristics of pipes 1, 2 and 3 can be determined using graphical methods or tables (Chow, 1959). For the full flow capacity of circular pipes, they may be calculated directly from Manning's equation using nomograms (McGhee, 1991) and semi-graphical methods (Zeghadnia et al., 2009), which make the computation process limited, tedious and inexact.

First Case: Watershed Arranged in Series Form
Flow in the storm sewer network is generally open channel flow. Most routine calculations in the design
or analysis of storm or sanitary sewer systems involve a condition called steady uniform flow, where the slope, cross-sectional flow area, and velocity are not time-related and are also constant in the length of pipe being analysed (Carlier, 1985). In this case, the Manning formula is the best model to describe and solve open channel problems and is written as:

$$
\begin{align*}
Q & =\frac{1}{n} R_{h}^{2 / 3} A S^{1 / 2}  \tag{3}\\
V & =\frac{1}{n} R_{h}^{2 / 3} A S^{1 / 2} \tag{4}
\end{align*}
$$

To describe flow in partially filled pipes, Equations (3) and (4) can be rewritten in terms of the water surface angle of the pipe, as shown in Figure 3, as follows:

$$
\begin{align*}
Q & =\frac{1}{n}\left(\frac{D^{8}}{2^{13}}\right)^{1 / 3}\left[\frac{(\theta-\sin \theta)^{5}}{\theta^{2}}\right]^{1 / 3} S^{1 / 2}  \tag{5}\\
V & =\frac{1}{n}\left(\frac{D}{4}\right)^{2 / 3}\left[\frac{(\theta-\sin \theta)^{5}}{\theta}\right]^{2 / 3} S^{1 / 2}  \tag{6}\\
R_{h} & =\frac{A}{P}=\frac{D}{4}\left(1-\frac{\sin (\theta)}{\theta}\right)  \tag{7}\\
A & =\frac{D^{2}}{8}(\theta-\sin (\theta))  \tag{8}\\
P & =\theta \frac{D}{2} \tag{9}
\end{align*}
$$

where $Q$ : flow rate in $\mathrm{m}^{3} / \mathrm{s} ; V$ : velocity of flow in $\mathrm{m} / \mathrm{s}$; $S$ : slope of pipe bottom, dimensionless; $n$ : channel roughness coefficient (Manning n ) in $\mathrm{m}^{-1 / 3} s ; R_{\mathrm{h}}$ : hydraulic radius of channel in $\mathrm{m} ; A$ : cross-sectional flow area in $\mathrm{m}^{2} ; P$ : wetted perimeter in $\mathrm{m} ; D$ : inner diameter in $m ; \theta$ : water surface angle in radian.

Equation (6) can be rewritten in the following form (Zeghadnia et al., 2009):

$$
\begin{equation*}
V=\left(\left(\frac{S^{1 / 2}}{n}\right)^{3}\left(\frac{2 Q}{D}\right)^{2}\right)^{1 / 5} \theta^{-2 / 5} \tag{10}
\end{equation*}
$$

Using Equation (10), Equations (7) and (8) can be rewritten as follows:

$$
\begin{align*}
R_{h} & =\left(\frac{n}{S^{0.5}}\right)^{3 / 5}\left(\frac{2 Q}{D \theta}\right)^{3 / 5}  \tag{11}\\
A & =\left(\frac{n}{S^{0.5}}\right)^{3 / 5}(2 Q)^{3 / 5}(D \theta)^{2 / 5} \tag{12}
\end{align*}
$$

where $\theta$ is water surface angle in radians as shown in Figure 3.


Figure 3: Water surface angle.

## Flow Velocity

In the case of a partially filled pipe and according to Equation (10), the previous formula can be rewritten as follows, where pipe 01 is the reference pipe (Zeghadnia et al., 2014b):

$$
\begin{equation*}
V_{3}=\left(\frac{Q_{3}}{Q_{1}}\right)^{1 / 4}\left(\frac{S_{3}^{0.5}}{n_{3}}\right)^{3 / 4}\left(\frac{n_{1}}{S_{1}^{0.5}}\right)^{3 / 20}\left(\frac{2 Q_{1}}{D_{1} \theta_{1}}\right)^{2 / 5} \tag{13}
\end{equation*}
$$

The flow velocity from Equation (6) can be calculated as a function of the reference pipe parameters. Equation (13) allows us to calculate the flow velocity in a partially filled pipe, but for a full pipe, the equation of the water surface angle is $\theta=2 \pi \theta=2 \pi$.

## Pipe Diameter

As reported by Zeghadnia (2014b), from Figure 1 it is straightforward to conclude that

$$
\begin{equation*}
Q_{3}>Q_{1} \tag{14}
\end{equation*}
$$

where:

$$
\begin{align*}
& Q_{3}=A_{3} V_{3}  \tag{15}\\
& Q_{1}=A_{1} V_{1} \tag{16}
\end{align*}
$$

From Equation (14) and for a full pipe (under atmospheric pressure), we obtain the following:

$$
\begin{equation*}
A_{3}=a A_{1} \tag{17}
\end{equation*}
$$

This gives:

$$
\begin{equation*}
D_{3}^{2}=a D_{1}^{2} \tag{18}
\end{equation*}
$$

For the full Manning equation, the flow velocity is written as follows:

$$
\begin{equation*}
V_{3}=\frac{s_{3}^{0.5}}{n_{3}}\left(\frac{D_{3}}{4}\right)^{2 / 3} \tag{19}
\end{equation*}
$$

If we substitute Equation (18) into (19), we obtain the following:

$$
\begin{equation*}
V_{3}=\frac{s_{3}^{0.5}}{n_{3}}\left(\frac{D_{1}}{4}\right)^{2 / 3} a^{1 / 3} \tag{20}
\end{equation*}
$$

This gives:

$$
\begin{equation*}
a=V_{3}^{3}\left(\frac{n_{3}}{s_{3}^{0.5}}\right)^{3}\left(\frac{4}{D_{1}}\right)^{2} \tag{21}
\end{equation*}
$$

Equation (21) is obtained from the full pipe case, and from Equations (18) and (21) we can conclude the following:

$$
\begin{equation*}
D_{3}=4 V_{3}^{3 / 2}\left(\frac{n_{3}}{s_{3}^{0.5}}\right)^{3 / 2} \tag{22}
\end{equation*}
$$

Inserting Equations (10) and (11) into (22) yields:

$$
\begin{equation*}
D_{3}=\left(\frac{Q_{3} n_{3}}{s_{3}^{0.5}}\right)^{3 / 8}\left(\frac{s_{1}^{0.5}}{Q_{1} n_{1}}\right)^{3 / 8} D_{1} \tag{23}
\end{equation*}
$$

The use of Equation (23) enables us to compute the pipe diameter as a function of the reference pipe parameters for both full and partially filled circular pipes.

## Accuracy Test

No error was recorded for the entire range of the water surface angle " $\theta$ " for $\theta_{3}=\theta_{1}$ as shown in Table 1 .

## Hydraulic Radius

From Equation (4), the hydraulic radius of the circular pipe can be written as follows:

$$
\begin{equation*}
R_{h 3}=V_{3}^{3 / 2}\left(\frac{n_{3}}{s_{3}^{0.5}}\right)^{3 / 2} \tag{24}
\end{equation*}
$$

To calculate the hydraulic radius in the case of a partially filled section (circular) as a function of the reference pipe parameters, we shall introduce the water surface angle by using Equation (13) as follows:

$$
\begin{equation*}
R_{h 3}=\left(\frac{Q_{3} n_{3}}{s_{3}^{0.5}}\right)^{3 / 8}\left(\frac{Q_{1} n_{1}}{s_{1}^{0.5}}\right)^{9 / 40}\left(\frac{2}{D_{1} \theta_{1}}\right)^{3 / 5} \tag{25}
\end{equation*}
$$

## Accuracy Test

Using Equation (25), no error was recorded for the

Table 1: Accuracy test of Equation (23) compared to Manning equation

| $\theta$ | Manning Equation (5) (m) | Proposed Equation (23) (m) | $A B S\left(\frac{D_{e q 23}-D_{e q 5}}{D_{e q 5}}\right) \%$ |
| :---: | :---: | :---: | :---: |
| $1^{\circ}$ | 1 | 1 | 0 |
| $2^{\circ}$ | 1 | 1 | 0 |
| $3^{\circ}$ | 1 | 1 | 0 |
| $4^{\circ}$ | 1 | 1 | 0 |
| $5^{\circ}$ | 1 | 1 | 0 |
| $6^{\circ}$ | 1 | 1 | 0 |
| $7^{\circ}$ | 1 | 1 | 0 |
| $8^{\circ}$ | 1 | 1 | 0 |
| $9^{\circ}$ | 1 | 1 | 0 |
| $10^{\circ}$ | 1 | 1 | 0 |
| $20^{\circ}$ | 1 | 1 | 0 |
| $25^{\circ}$ | 1 | 1 | 0 |
| $35^{\circ}$ | 1 | 1 | 0 |
| $45^{\circ}$ | 1 | 1 | 0 |
| $90^{\circ}$ | 1 | 1 | 0 |
| $110^{\circ}$ | 1 | 1 | 0 |
| $120^{\circ}$ | 1 | 1 | 0 |
| $135^{\circ}$ | 1 | 1 | 0 |
| $145^{\circ}$ | 1 | 1 | 0 |
| $190^{\circ}$ | 1 | 1 | 0 |
| $200^{\circ}$ | 1 | 1 | 0 |
| $235^{\circ}$ | 1 | 1 | 0 |
| $245^{\circ}$ | 1 | 1 | 0 |
| $246^{\circ}$ | 1 | 1 | 0 |
| $290^{\circ}$ | 1 | 1 | 0 |
| $300^{\circ}$ | 1 | 1 | 0 |
| $308^{\circ}$ | 1 | 1 | 0 |
| $320^{\circ}$ | 1 | 1 | 0 |
| $335^{\circ}$ | 1 | 1 | 0 |
| $345^{\circ}$ | 1 | 1 | 0 |
| $350^{\circ}$ | 1 | 1 | 0 |
| $360^{\circ}$ | 1 | 1 | 0 |

entire range of the water surface angle " $\theta$ " when $\theta_{3}=\theta_{1}$, as shown in Table 2.

## Second Case: Watershed Arranged <br> in Parallel Form

As shown in Figure 2, the study can be divided into four possible types of problems to compute the characteristics of the pipes in relation to the reference pipe:

Table 2: Accuracy test of Equation (25) compared to Manning equation

| $\theta$ | Manning <br> Equation $(7)(m)$ | Proposed <br> Equation $(25)(m)$ | $A B S\left(\frac{R h_{e q 25}-R h_{\text {eq } 7}}{R h_{\text {eq } 7}}\right) \%$ |
| :---: | :---: | :---: | :---: |
| $1^{\circ}$ | $1,267933.10^{-05}$ | $1,267933.10^{-05}$ | 0 |
| $2^{\circ}$ | $5,071502.10^{-05}$ | $5,071502.10^{-05}$ | 0 |
| $3^{\circ}$ | $1,141001.10^{-04}$ | $1,141001.10^{-04}$ | 0 |
| $4^{\circ}$ | $2,028231.10^{-04}$ | $2,028231.10^{-04}$ | 0 |
| $5^{\circ}$ | $3,168676.10^{-04}$ | $3,168676.10^{-04}$ | 0 |
| $6^{\circ}$ | $4,56213.10^{-04}$ | $4,56213.10^{-04}$ | 0 |
| $7^{\circ}$ | $6,208338.10^{-04}$ | $6,208338.10^{-04}$ | 0 |
| $8^{\circ}$ | $8,106999.10^{-04}$ | $8,106999.10^{-04}$ | 0 |
| $9^{\circ}$ | $1,025777.10^{-03}$ | $1,025777.10^{-03}$ | 0 |
| $10^{\circ}$ | $1,266025.10^{-03}$ | $1,266025.10^{-03}$ | 0 |
| $20^{\circ}$ | $5,041032.10^{-03}$ | $5,041032.10^{-03}$ | 0 |
| $25^{\circ}$ | $7,849684.10^{-03}$ | $7,849684.10^{-03}$ | 0 |
| $35^{\circ}$ | $1,125632.10^{-02}$ | $1,125632.10^{-02}$ | 0 |
| $45^{\circ}$ | $1,524547.10^{-02}$ | $1,524547.10^{-02}$ | 0 |
| $90^{\circ}$ | $9,076439.10^{-02}$ | $9,076439.10^{-02}$ | 0 |
| $110^{\circ}$ | .12753 | .12753 | 0 |
| $120^{\circ}$ | .14651 | .14651 | 0 |
| $135^{\circ}$ | .174846 | .174846 | 0 |
| $145^{\circ}$ | .1932062 | .1932062 | 0 |
| $190^{\circ}$ | .262973 | .262973 | 0 |
| $200^{\circ}$ | .2743886 | .2743886 | 0 |
| $235^{\circ}$ | .2998822 | .2998822 | 0 |
| $245^{\circ}$ | .3029605 | .3029605 | 0 |
| $246^{\circ}$ | .3031687 | .3031687 | 0 |
| $290^{\circ}$ | .2964809 | .2964809 | 0 |
| $300^{\circ}$ | .2914339 | .2914339 | 0 |
| $308^{\circ}$ | .286744 | .286744 | 0 |
| $320^{\circ}$ | .2788843 | .2788843 | 0 |
| $335^{\circ}$ | .2681943 | .2681943 | 0 |
| $345^{\circ}$ | .2608737 | .2508737 | 0 |
| $350^{\circ}$ | .2572351 | 0501268 | 0 |
| $360^{\circ}$ |  |  | 0 |
|  |  |  | 0 |

1. The computation of pipe 3 as a function of the pipe 1 characteristics (pipe 1 is the reference pipe).
2. The computation of pipe 3 as a function of the pipe 2 characteristics (pipe 2 is the reference pipe).
3. The computation of pipe 2 as a function of the pipe 1 characteristics (pipe 1 is the reference pipe).
4. The computation of pipe 1 as a function of the pipe 2 characteristics (pipe 2 is the reference pipe).

## Types One and Two

Flow Velocity
Let us consider that pipe M1-N1 (or pipe 1) is the reference pipe, with known parameters. As such, the diameter $D_{2}$, hydraulic radius $R_{h 2}$, surface water angle $\theta_{2}$, cross-sectional water flow $A_{2}$, slope $S_{2}$, are known data. The slope $S_{3}$ and roughness $n_{3}$ are considered as known parameters for pipes N1-N2 (or pipe 3). In the
case where pipe M1-N1 is the reference pipe, the flow velocity can be calculated as follows (Zeghadnia et al., 2014c):

$$
\begin{equation*}
V_{3}=\left(\frac{Q_{3}}{Q_{1}}\right)^{1 / 4}\left(\frac{s_{3}^{0.5}}{n_{3}}\right)^{3 / 4}\left(\frac{n_{1}}{s_{1}^{0.5}}\right)^{3 / 20}\left(\frac{2 Q_{1}}{D_{1} \theta_{1}}\right)^{2 / 5} \tag{26}
\end{equation*}
$$

If pipe 2 is the reference pipe, then previous formula can be rewritten as follows:

$$
\begin{equation*}
V_{3}=\left(\frac{Q_{3}}{Q_{2}}\right)^{1 / 4}\left(\frac{s_{3}^{0.5}}{n_{3}}\right)^{3 / 4}\left(\frac{n_{2}}{s_{2}^{0.5}}\right)^{3 / 20}\left(\frac{2 Q_{2}}{D_{2} \theta_{2}}\right)^{2 / 5} \tag{27}
\end{equation*}
$$

## Types Three and Four

Here, we will write the characteristics of the first pipe using the characteristics of the second pipe (reference pipe), as will be shown in the following sections. As reported by Zeghadnia et al. (2014c), the equations can be written as follows:

$$
\begin{equation*}
V_{1}=\left(\frac{Q_{1}}{Q_{2}}\right)^{1 / 4}\left(\frac{s_{1}^{0.5}}{n_{1}}\right)^{3 / 4}\left(\frac{n_{2}}{s_{2}^{0.5}}\right)^{3 / 20}\left(\frac{2 Q_{2}}{D_{2} \theta_{2}}\right)^{2 / 5} \tag{28}
\end{equation*}
$$

For the case when pipe 1 is the reference pipe, the same result can be obtained as follows:

$$
\begin{equation*}
V_{2}=\left(\frac{Q_{2}}{Q_{1}}\right)^{1 / 4}\left(\frac{s_{2}^{0.5}}{n_{2}}\right)^{3 / 4}\left(\frac{n_{1}}{s_{1}^{0.5}}\right)^{3 / 20}\left(\frac{2 Q_{1}}{D_{1} \theta_{1}}\right)^{2 / 5} \tag{29}
\end{equation*}
$$

## Pipe Diameter

To obtain the equations of the pipe diameter and hydraulic radius for the four possible types of problems as a function of the reference pipe parameters (as shown in Figure 2), we follow the same steps as shown above.

For the case of types three and four, as described above, when pipe 1 is the reference pipe, the same form of results can be obtained. As reported by Zeghadnia (2014c), from Figure 1 it is straightforward to conclude:

$$
\begin{align*}
& Q_{3}>Q_{1}  \tag{30}\\
& Q_{3}>Q_{2} \tag{31}
\end{align*}
$$

From Equations (26) and (27) and for a full pipe (under atmospheric pressure), we obtain the following:

$$
\begin{align*}
& A_{3}=t A_{1}  \tag{32}\\
& A_{3}=b A_{2} \tag{33}
\end{align*}
$$

This gives:

$$
\begin{equation*}
D_{3}^{2}=t D_{1}^{2} \tag{34}
\end{equation*}
$$

$$
\begin{equation*}
D_{3}^{2}=b D_{2}^{2} \tag{35}
\end{equation*}
$$

According to Zeghadnia et al. (2014c), the coefficients " $t$ " and " $b$ " can be written as follows:

$$
\begin{align*}
& t=V_{3}^{3}\left(\frac{n_{3}}{s_{3}^{0.5}}\right)^{3}\left(\frac{4}{D_{1}}\right)^{2}  \tag{36}\\
& b=V_{3}^{3}\left(\frac{n_{3}}{s_{3}^{0.5}}\right)^{3}\left(\frac{4}{D_{2}}\right)^{2} \tag{37}
\end{align*}
$$

From Equations (11), (13), (34) and (36), we can derive the following:

$$
\begin{equation*}
D_{3}=\left(\frac{Q_{3} n_{3}}{S_{3}^{0.5}}\right)^{3 / 8}\left(\frac{S_{1}^{0.5}}{Q_{1} n_{1}}\right)^{3 / 8} D_{1} \tag{38}
\end{equation*}
$$

The use of Equation (38) enables us to compute the pipe diameter as a function of the reference pipe parameters, for both full and partially filled circular pipes.

Similarly, in the case where pipe M2-N1 is the reference pipe, the pipe diameter can be calculated using Equation (39):

$$
\begin{equation*}
D_{3}=\left(\frac{Q_{3} n_{3}}{S_{3}^{0.5}}\right)^{3 / 8}\left(\frac{S_{2}^{0.5}}{Q_{2} n_{2}}\right)^{3 / 8} D_{2} \tag{39}
\end{equation*}
$$

For types three and four, let us consider that pipe 2 is the reference pipe with known parameters. Using Equations (34) and (35), we obtain the following:

$$
\begin{array}{r}
t D_{1}^{2}=b D_{2}^{2} \\
D_{1}=\left(\frac{b}{t}\right)^{0.5} D_{2} \tag{41}
\end{array}
$$

Based on Equations (4) and (41), we obtain:

$$
\begin{equation*}
\frac{t}{b}=\frac{1}{V_{1}^{3}}\left(\frac{s_{1}^{0.5}}{n_{1}}\right)^{3}\left(\frac{D_{2}}{4}\right)^{2} \tag{42}
\end{equation*}
$$

Also, from Equations (30), (31), (32) and (33) we can conclude the following:

$$
\begin{equation*}
\frac{t}{b}=\frac{V_{1}}{V_{2}}\left(\frac{Q_{2}}{Q_{1}}\right) \tag{43}
\end{equation*}
$$

where:

$$
\begin{equation*}
b=\left(\frac{Q_{3}}{Q_{2}}\right)^{3 / 4}\left(V_{2}\right)^{3 / 4}\left(\frac{n_{3}}{S_{3}^{0.5}}\right)^{3 / 4}\left(\frac{4}{D_{2}}\right)^{1 / 2} \tag{44}
\end{equation*}
$$

Using Equations (11), (28), (42) and (44), whatever the case-full or partially filled pipe-the diameter equation can be written as follows:

$$
\begin{equation*}
D_{1}=\left(\frac{Q_{1} n_{1}}{S_{1}^{0.5}}\right)^{3 / 8}\left(\frac{S_{2}^{0.5}}{Q_{2} n_{2}}\right)^{3 / 8} D_{2} \tag{45}
\end{equation*}
$$

Similarly, in the case where pipe 01 is the reference pipe, for the case of full or partially filled pipes the diameter can be calculated as follows:

$$
\begin{equation*}
D_{2}=\left(\frac{Q_{2} n_{2}}{S_{2}^{0.5}}\right)^{3 / 8}\left(\frac{S_{1}^{0.5}}{Q_{1} n_{1}}\right)^{3 / 8} D_{1} \tag{46}
\end{equation*}
$$

Accuracy Test
From the assessment of the accuracy of Equation (45) -which gives the same accuracy as Equation (46) -as presented in Table 3, no error was recorded for the entire range of the water surface angle " $\theta$ " when $\theta_{2}=\theta_{1}$.

## Hydraulic Radius

Starting with the case when pipe 2 is the reference pipe, by combining Equations (4) and (28), we can get the following equation:

$$
\begin{equation*}
R_{h 1}=\left(\frac{Q_{1} n_{1}}{S_{1}^{0.5}}\right)^{3 / 8}\left(\frac{Q_{2} n_{2}}{S_{2}^{0.5}}\right)^{9 / 40}\left(\frac{2}{D_{2} \theta_{2}}\right)^{3 / 5} \tag{47}
\end{equation*}
$$

Similarly, for the case when pipe 1 is the reference pipe, the expression of the hydraulic radius $R_{h 2}$ in pipe 2 can be written as follows:

$$
\begin{equation*}
R_{h 1}=\left(\frac{Q_{2} n_{2}}{S_{2}^{0.5}}\right)^{3 / 8}\left(\frac{Q_{1} n_{1}}{S_{1}^{0.5}}\right)^{9 / 40}\left(\frac{2}{D_{1} \theta_{1}}\right)^{3 / 5} \tag{48}
\end{equation*}
$$

## Accuracy Test

The proposed approach using Equation (47) -even for Equation (48) —produces an exact solution for all $\theta$ values, as shown in Table 4. The maximum error recorded is almost zero: $1,19 \cdot 10^{-5}$ when $\theta_{1}=\theta_{2}$.

## Computing of Storm Sewer Network

The design of the storm sewer network can be worked out once the following steps are taken:
Step 1: Define the storm sewer system type,
Step 2: Define the reference pipe,
Step 3: Calculate the new diameter as a function of the reference pipe,
Step 4: Calculate the velocity as a function of the reference pipe,

Table 3: Accuracy test of the equation (45) compared to Manning equation.

| $\theta$ | Manning Equation (5) (m) | Proposed <br> Equation <br> (45)(m) | $A B S\left(\frac{D_{e q 45}-D_{e q 5}}{D_{e q 5}}\right) \%$ |
| :---: | :---: | :---: | :---: |
| $1^{\circ}$ | 0.5 | 0.5 | 0 |
| $2^{\circ}$ | 0.5 | 0.5 | 0 |
| $3^{\circ}$ | 0.5 | 0.5 | 0 |
| $4^{\circ}$ | 0.5 | 0.5 | 0 |
| $5^{\circ}$ | 0.5 | 0.5 | 0 |
| $6^{\circ}$ | 0.5 | 0.5 | 0 |
| $7^{\circ}$ | 0.5 | 0.5 | 0 |
| $8^{\circ}$ | 0.5 | 0.5 | 0 |
| $9^{\circ}$ | 0.5 | 0.5 | 0 |
| $10^{\circ}$ | 0.5 | 0.5 | 0 |
| $20^{\circ}$ | 0.5 | 0.5 | 0 |
| $25^{\circ}$ | 0.5 | 0.5 | 0 |
| $35^{\circ}$ | 0.5 | 0.5 | 0 |
| $45^{\circ}$ | 0.5 | 0.5 | 0 |
| $90^{\circ}$ | 0.5 | 0.5 | 0 |
| $110^{\circ}$ | 0.5 | 0.5 | 0 |
| $120^{\circ}$ | 0.5 | 0.5 | 0 |
| $135^{\circ}$ | 0.5 | 0.5 | 0 |
| $145^{\circ}$ | 0.5 | 0.5 | 0 |
| $190^{\circ}$ | 0.5 | 0.5 | 0 |
| $200^{\circ}$ | 0.5 | 0.5 | 0 |
| $235^{\circ}$ | 0.5 | 0.5 | 0 |
| $245^{\circ}$ | 0.5 | 0.5 | 0 |
| $246{ }^{\circ}$ | 0.5 | 0.5 | 0 |
| $290^{\circ}$ | 0.5 | 0.5 | 0 |
| $300^{\circ}$ | 0.5 | 0.5 | 0 |
| $308{ }^{\circ}$ | 0.5 | 0.5 | 0 |
| $320^{\circ}$ | 0.5 | 0.5 | 0 |
| $335^{\circ}$ | 0.5 | 0.5 | 0 |
| $345^{\circ}$ | 0.5 | 0.5 | 0 |
| $350^{\circ}$ | 0.5 | 0.5 | 0 |
| $360^{\circ}$ | 0.5 | 0.5 | 0 |

Step 5: Calculate the hydraulic radius as a function of the reference pipe or use the results of steps 3 and 4 to conclude the result.

Table 4: Accuracy test of Equation (47) compared to Manning equation

| $\theta$ | Manning Equation(7) (m) | Proposed Equation(47) (m) | $A B S\left(\frac{R h_{e q 47}-R h_{e q 7}}{R h_{e q 7}}\right) \%$ |
| :---: | :---: | :---: | :---: |
| $1^{\circ}$ | 6,339667.10-06 | 6,339667.10-06 | 0 |
| $2^{\circ}$ | $2,535751.10^{-05}$ | 2,535751.10-05 | 7,173376.10-06 |
| $3^{\circ}$ | 5,705006.10-05 | 5,705006.10-05 | 0 |
| $4^{\circ}$ | 1,014115.10-04 | 1,014115.10-04 | 7,174685.10-06 |
| $5^{\circ}$ | 1,584338.10-04 | 1,584338.10-04 | 0 |
| $6^{\circ}$ | 2,281065.10-04 | 2,281065.10-04 | 0 |
| $7^{\circ}$ | 3,104169.10-04 | 3,104169.10-04 | 0 |
| $8^{\circ}$ | 4,053499.10-04 | 4,053499.10-04 | 0 |
| $9^{\circ}$ | $5,128883.10^{-04}$ | 5,128884.10-04 | 1,134899.10-05 |
| $10^{\circ}$ | $6.330123 .10^{-04}$ | 6,330124.10-04 | 9,195344.10-06 |
| $20^{\circ}$ | 2,520516.10-03 | 2,520516.10-03 | 0 |
| $25^{\circ}$ | 3,924842.10-03 | 3,924842.10-03 | 0 |
| $35^{\circ}$ | 7,622734.10-03 | 7,622734.10-03 | 0 |
| $45^{\circ}$ | 1,244822.10-02 | 1,244822.10-02 | 7,481571.10-06 |
| $90^{\circ}$ | 4,538219.10-02 | 4,538219.10-02 | 0 |
| $110^{\circ}$ | 6,376502.10-02 | 6,376502.10-02 | 0 |
| $120^{\circ}$ | 7,325502.10-02 | 7,325502.10-02 | 1,017074.10-05 |
| $135^{\circ}$ | $8,742299.10^{-02}$ | $8,742299.10^{-02}$ | 0 |
| $145^{\circ}$ | . 0966031 | 9,660311.10-02 | 7,712569.10-06 |
| $190^{\circ}$ | . 1314865 | . 1314865 | 0 |
| $200^{\circ}$ | . 1371943 | . 1371943 | 1,086135.10-05 |
| $235^{\circ}$ | . 1499411 | . 1499411 | 9,938009.10-06 |
| $245^{\circ}$ | . 1514803 | . 1514803 | 9,837031.10-06 |
| $246{ }^{\circ}$ | . 1515843 | . 1515843 | 9,830278.10-06 |
| $290{ }^{\circ}$ | . 1482405 | . 1482404 | 1,005202.10-05 |
| $300^{\circ}$ | . 145717 | . 145717 | 0 |
| $308^{\circ}$ | . 143372 | . 143372 | 0 |
| $320^{\circ}$ | . 1394421 | . 1394421 | 0 |
| $335^{\circ}$ | . 1340972 | . 1340972 | 0 |
| $346{ }^{\circ}$ | . 1300717 | . 1300717 | 1,145612.10-05 |
| $350{ }^{\circ}$ | . 1286175 | . 1286176 | 1,158564.10-05 |
| $360{ }^{\circ}$ | . 1250634 | . 1250634 | 1,191489.10 ${ }^{-05}$ |

## Conclusion

In this study, we present a new method to calculate a storm sewer network as a function of a known reference pipe (KRPM), and this is applicable for two cases according to the arrangement of the watershed drainage system (in series and parallel form), with both full and partially filled pipes. Computing the flow velocity, pipe
diameter and hydraulic radius becomes easy, directly for all these parameters and in different cases with equal filling rates to that of the reference pipes. Thus, it is clear that there is a relationship between the pipes parameters and the flow in the up-pipe, which affects the parameters of the down-pipe, as explained above for both full and partially filled pipes.

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