# Estimation of the empirical model parameters of unsaturated soils

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**Abstract.** For each flow modelling in the unsaturated soils, it is necessary to determine the retention curve and the hydraulic conductivity curve of studied soils. Some empirical models use the same parameters to describe these two hydraulic properties. For this reason, the estimation of these parameters is achieved by adjusting the experimental points to the retention curve only, which is more easily measured as compared with the hydraulic conductivity curve. In this work, we show that the adjustment of the retention curve θ (h) is not generally sufficient to describe the hydraulic conductivity curve K (θ) and the spatio-temporal variation of the moisture in the soil θ (z). The models used in this study are van Genuchten- Mualem model (1980-1976) and Brooks and Corey model (1964), for two different soils; Gault clay and Givors silt.

## 1 Introduction

In practice, the retention curve θ (h) is easy to measure compared to the hydraulic conductivity curve K (θ). Therefore, some formulations, based on statistical pore-size distribution methods, have been proposed to predict the unsaturated hydraulic conductivity function K (θ) from knowledge the retention curve θ (h). Authors who have adopted this approach are numerous (Childs & Collis-Georges, Burdine, Marshall, Campbell, Mualem, Fredlund and Xing) [1-2-3-4-5-6]. The most widely models used are Burdine (1953) and Mualem (1976). Among a large number of the retention curve models proposed, only few can easily incorporate into these pore-size distribution models such as the functions proposed by Brooks and Corey, Brutsaert and van Genuchten [7-8-9]. Brooks and Corey used the Burdine model to predict K (θ), for against van Genuchten used Mualem model, usually noted van Genuchten-Mualem model.

For this reason, these empirical models estimate the hydrodynamic properties θ (h) and K(θ) using the same parameters, These parameters are obtained usually by fitting the experimental points of the retention curve θ (h) only, the hydraulic conductivity curve K (θ) is deduced after.

The aim of this work is to verify if the calculated hydraulic conductivity curve with these adjusted parameters can describe the measured one. And can also provide the spatio-temporal variation of the moisture in the soil θ (z). The models used in this study are the combined model of van Genuchten-Mualem (1980-1976). and Brooks and Corey model (1964). The choice of models is based on a comparative study conducted by Sillers [10] cited by Fredlund and Houston [11]. This choice depends also on the difference between the expressions of the models, their popularity and their use in the literature. This study was carried on two different soils: Gault clay and Givors silt.

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## 2 hydraulic properties

The expressions of the water retention characteristics curve θ (h) and the hydraulic conductivity curve K (θ) of the used models in this work are defined as follows:

### 2.1 van Genuchten-Mualem model

The combined model of the hydraulic conductivity and retention curve van Genuchten-Mualem (1980-1976) is currently the most used model. Many authors have considered it as appropriate to a large range of soil, especially for fine soils [12-13]. This choice model takes also into consideration the strong nonlinearity of the hydrodynamic properties.

$$\theta_e = \frac{\theta - \theta_r}{\theta_s - \theta_r} = [1 + (\alpha h)^n]^{-m} \qquad (1)$$

$$K(\theta) = K_s \left(\frac{\theta - \theta_r}{\theta_s - \theta_r}\right)^{0.5} [1 - [1 - (\frac{\theta - \theta_r}{\theta_s - \theta_r})^{\frac{1}{m}}]^m]^2 \qquad (2)$$

Where: $\theta_e$= normalized volumetric water content; $\theta_s$= volumetric water content at saturation $[L^3/L^3]$; $\theta_r$= residual volumetric water content $[L^3/L^3]$; $K_s$= hydraulic conductivity at saturation $[L.T^{-1}]$. α = parameter related to the entry air pressure $[L^{-1}]$; n=a dimensionless coefficient related to the pore size distribution with n≥1; m= Mualem coefficient defined by: m =1-1/n [5].

### 2.2 Brooks and Corey model

The simplicity of the expression of the Brooks and Corey model (1964) made that it is often used in numerical models to study unsaturated media; it is based on the assumption of the existence of the air entry pressure. Brooks and Corey used the Burdine model to predict hydraulic conductivity. θ (h) and K (θ) are written as follows:

$$\theta = \theta_s \text{ For } h \le h_{ae} \quad (3a)$$

$$\theta_e(h) = \frac{\theta - \theta_r}{\theta_s - \theta_r} = \left(\frac{h_{ae}}{h}\right)^N \text{ For } h > h_{ae} \quad (3b)$$

$$K(\theta) = K_s \left(\frac{\theta - \theta_r}{\theta_s - \theta_r}\right)^{\frac{M}{N}} \quad (4)$$

Where: $\theta_e$=normalized volumetric water content; $\theta_s$= volumetric water content at saturation [$L^3/L^3$]; $\theta_r$= residual volumetric water content [$L^3/L^3$]; $K_s$= hydraulic conductivity at saturation. [$L.T^{-1}$]; $h_{ae}$ =the air entry pressure [L]; N = an empirical parameter often referred to as the pore size distribution index; M= a constant defined as M=2+3N [2].

### 2.3 Geotechnical characteristics of the tested soils

The tested soils in this study are the Gault clay and Givors silt. The geotechnical characteristics of these soils determined by Bentoumi [14-15] are presented in the table 1:

**Table 1.** Soil Geotechnical Characteristics

| Properties | Gault clay | Givors silt |
|---|---|---|
| % element < 80µm | 100 | 92 |
| % element < 2µm | 69 | 15 |
| LL (%) | 40 | 33 |
| WL (%) | 19 | 23 |
| IP (%) | 21 | 10 |
| $W_{opt}$ (%) | 17.5 | 16.3 |
| $K_s$(cm/mn) | 1.5 $10^{-6}$ | 1.2 $10^{-4}$ |
| $\theta_s$ ($cm^3/cm^3$) | 0.365 | 0.355 |
| $\theta_r$ ($cm^3/cm^3$) | 0.125 | 0.025 |
| ($\gamma_d/\gamma_w$) | 1.77 | 1.73 |

## 3 Estimation of the parameters

### 3.1 Retention curve

Estimated parameter values for the studied soils are listed in tables 2 and 3, for van Genuchten-Mualem and Brooks and Corey empirical models respectively. The parameters values are obtained by fitting the models (equation 1 and 3) to the measured points of the retention curve θ (h) [14-16] using Curve Expert software _1.3. In general, the correlation coefficient will range from 0 to 1, with a correlation coefficient of 1 being the best. But in some peculiar circumstances, Curve Expert gives (r) greater that one, which an unrealistic values. This is indicative of a very poor data model.

From table 2 and 3, the correlation coefficient values (r) reflect the good accuracy of the retention model parameters in describing observed data.

**Table 2.** Values of van Genuchten-Mualem model parameters adjusted from θ (h)

| Parameters | Gault clay | Givors silt |
|---|---|---|
| α ($cm^{-1}$) | 0.001975 | 0.002987 |
| n | 1.163 | 1.2845 |
| m | 0.149 | 0.221 |
| r | 0.971 | 0.9726 |

**Table 3.** Values of Brooks and Corey model parameters adjusted from θ (h)

| Parameters | Gault clay | Givors silt |
|---|---|---|
| $h_{ae}$ (cm) | 190.54 | 77.446 |
| N | 0.0998 | 0.1498 |
| M | 2.299 | 2.449 |
| r | 0.978 | 0.989 |

### 3.2 Hydraulic conductivity curve

The calculated hydraulic conductivity curves obtained by the empirical model (equation 2 and 4), using the adjusted parameters listed in table 2 and 3 respectively for van Genuchten-Mualem model and Brooks & Corey model, are compared with the measured hydraulic conductivity curve determined by the instantaneous profiles method (Indirect measurement) [14-15]. The purpose of this comparison is to verify if the calculated k(θ), using equation 2 and 4, can describe the measured one, and if the estimation of the parameters by fitting the measured θ(h) only is sufficient. This comparison is done by determining the correlation coefficient, as is shown in tables 4 and 5 respectively for van Genuchten-Mualem model and Brooks & Corey model. These measured and calculated curves for the both models are shown in figure 1 for Gault clay, and in figure 2 for the Givors silt.

**Table 4.** Correlation coefficient value obtained by comparing the calculated and measured K(θ) in case of van Genuchten-Mualem model

| Parameters | Gault clay | Givors silt |
|---|---|---|
| Measured $K_s$ (cm/mn) | 1.5 $10^{-6}$ | 1.2 $10^{-4}$ |
| m | 0.149 | 0.221 |
| r | - | - |

**Table 5.** Correlation coefficient value obtained by comparing the calculated and measured K (θ) in case of Brooks and Corey model

| Parameters | Gault clay | Givors silt |
|---|---|---|
| Measured $K_s$ (cm/mn) | 1.5 $10^{-6}$ | 1.2 $10^{-4}$ |
| N | 0.0998 | 0.1498 |
| M | 2.299 | 2.449 |
| r | - | - |

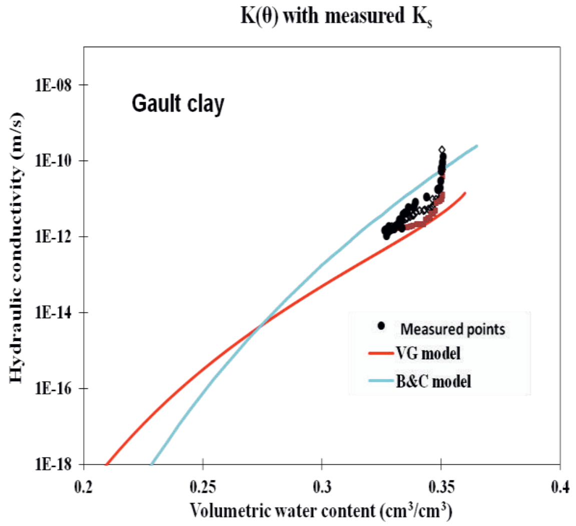


**Figure 1.** Hydraulic conductivity curves calculated with the measured value of $K_s$ compared to the measured one for Gault clay ,

From tables 4 and 5 and from figure 1 and 2, it is found that the fitted parameters values of van Genuchten-Mualem and Brooks and Corey models from the retention curve cannot represent fairly the experimental hydraulic conductivity curve for the both soils. No (r) value was obtained in each case, which indicates no agreement between the calculated and measured curves of K (θ).

Because of the high difference observed, two adjustments are performed. The first one is to determine the parameters values of the empirical models by fitting the hydraulic conductivity curve. In the second one, a new value of $K_s$ is estimated from measured hydraulic conductivity curve, by conserving the parameters adjusted from the water retention curve.

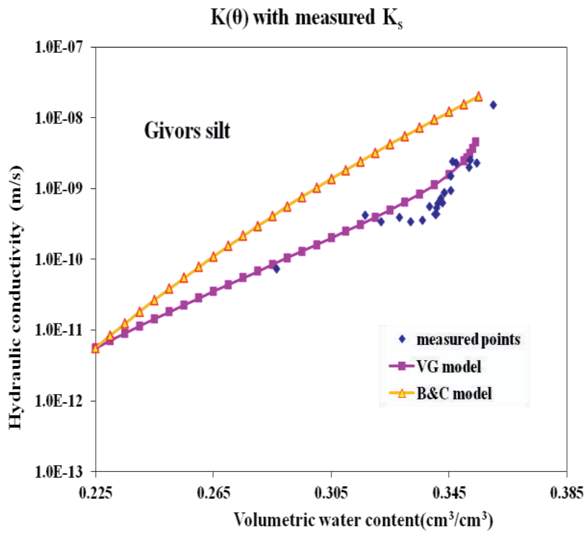


**Figure 2.** Hydraulic conductivity curves calculated with the measured value of $K_s$ compared to the measured one for Givors silt.

### *3.2.1 Estimation of the model parameters by adjusting K (θ) keeping measured $K_s$.*

The first adjustment of the hydraulic conductivity curve is to keep the value of $K_s$ and to seek a new value of (n) for van Genuchten-Mualem model, and (N) for the Brooks & Corey model. Subsequently a comparison of the measured retention curve and that calculated with these new values is performed. The results of this adjustment are shown in tables 6 and 7.

**Table 6.** New values of van Genuchten-Mualem model parameters adjusted from K (θ) using measured $K_s$

| Soils | n | m | r |
|---|---|---|---|
| **Gault clay** | 0.456 | 0.313 | 0.363 |
| **Givors silt** | 1.358 | 0.264 | 0.668 |

**Table 7.** New values of Brooks and Corey model parameters adjusted from K (θ) using measured $K_s$

| Soils | N | M | r |
|---|---|---|---|
| **Gault clay** | 2.92 | 9.76 | 1.67 |
| **Givors silt** | 3.734 | 13.202 | 2.93 |

The correlation coefficients values obtained by fitting the measured hydraulic conductivity curves K (θ) are low. For van Genuchten-Mualem model they are of the order of 0.363 for Gault clay and of 0.668 for Givors silt. For Brooks & Corey model, the correlation coefficients values obtained for the two soils are greater than 1, this is indicative of a very poor data model. In addition, this adjustment does not determine the values of α and $h_{ae}$. So, the retention curve θ (h) of each model can't be defined. This leads us to not accept the found values.

### *3.2.2 Estimation of a new value of $K_s$ keeping the adjusted parameter from θ (h).*

In the second adjustment, and knowing that the permeability value $K_s$ is obtained by the instantaneous profiles method (indirect measurement method), we try to find the best value of $K_s$ which can give a good correlation of the two curves of K(θ) (measured and calculated), keeping the parameters values obtained by adjusting the retention curve θ(h) .

**Table 8.** Calculated values $K_s$ for van Genuchten-Mualem model

| Soils | Calculated $K_s$ (cm/mn) | r |
|---|---|---|
| Gault clay | $1.914\ 10^{-5}$ | 0.630 |
| Givors silt | $7.98\ 10^{-5}$ | 0.818 |

**Table 9.** Calculated values $K_s$ for Brooks and Corey model

| Soils | Calculated $K_s$ (cm/mn) | r |
|---|---|---|
| Gault clay | $1.92\ 10^{-6}$ | 0.668 |
| Givors silt | $1.26\ 10^{-5}$ | 0.770 |

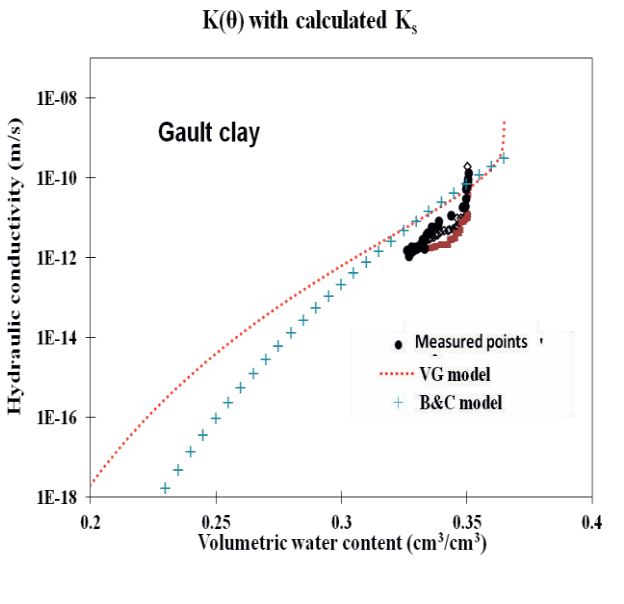


**Figure 3.** Hydraulic conductivity curves calculated with the new value of $K_s$ for Gault clay.

Figure 3 and tables 8 and 9 show that for the Gault clay, the result of the second adjustment is not satisfactory; the correlation coefficients values are of the order of 0.630 for the van Genuchten-Mualem model and $K_s$ is equal to 1.914 $10^{-5}$ (cm/min), and of the order of 0.668 for Brooks & Corey model and $K_s$ is equal to 1.92 $10^{-6}$ (cm/min). The new estimated value of $K_s$ is obtained with a low correlation coefficient because of few measured points used. Indeed the volumetric water content ranges from $\theta_i$=0.325($cm^3/cm^3$) to $\theta_s$=0.365 ($cm^3/cm^3$), when $\theta_r$= 0.125($cm^3/cm^3$).

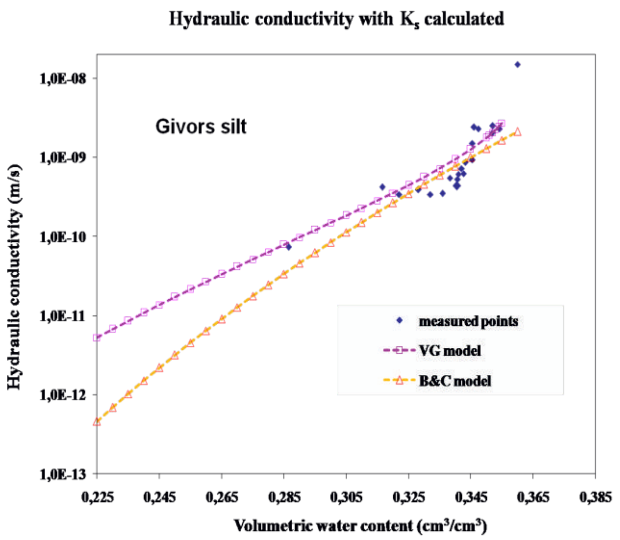


**Figure 4.** Hydraulic conductivity curves calculated with the new value of $K_s$ for Givors silt.

But for the Givors silt a good correlation is obtained with the new estimated value of $K_s$ (tables 8 and 9). They are of the order of 0.818 for the van Genuchten-Mualem model, and of 0.770 for the Brooks & Corey model. Figure 4 shows a clear improvement of the calculated curves comparing to the figure 2.

## 4 hydraulic profiles

Hydraulic profiles θ (z,t) present the spatio-temporal variation of the moisture in the soil. θ (z,t) are determined by the resolution of the Richards equation given by:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}[K(\theta)\frac{\partial}{\partial z}(h+z)] \quad (5)$$

Where: θ: soil volumetric water content [$L^3/L^3$], t: time [T], K: hydraulic conductivity [L/T], h: the water pressure head [L], z: the depth [L].

In this study we use the numerical model developed by Bouchemella [16-17] based on resolution of capacitive form of Richards's equation, which is written as follows:

$$C(h)\frac{\partial h}{\partial t} = \frac{\partial}{\partial z}(K(h)\frac{\partial h}{\partial z} + K(h)) \quad (6)$$

Where: $C(h) = \partial\theta / \partial h$ is the specific soil water content capacity [$L^{-1}$], (h> 0 is a suction)

To solve equation (6), θ (h) and K (θ) are defined using the parameter values adjusted from measured θ (h) and by using also, the measured value of $K_s$ for Gault clay and estimated value of $K_s$ for Givors silt. In order to test the impact of the choice of the fitting method on describing the hydraulic profiles θ (z,t). In this section only the van Genuchten- Mualem model is used.

### 4.1 Gault clay

The tested problem is a vertical infiltration simulation conducted on 25 cm long soil column. The flow domain is a homogeneous Gault clay layer. van Genuchten-Mualem empirical model is used, with the parameters

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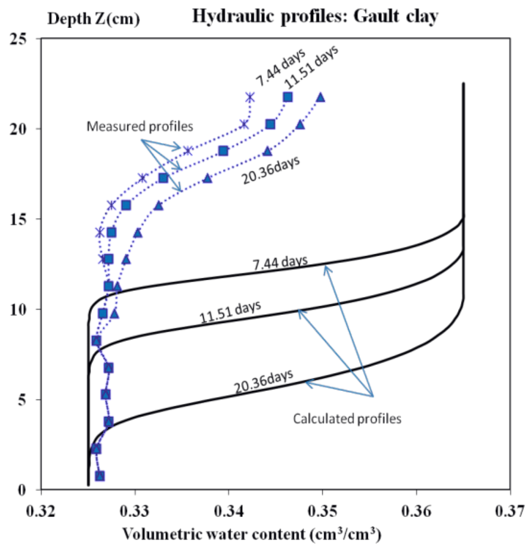


**Figure 5.** Hydraulic profiles of Gault clay

values listed in table 2 and the measured value of $K_s$. A water head pressure ($h_0$=-100 cm) is imposed at the top of the column combined with a zero flux at the bottom of the column. The soil was initially assumed in wetted a state with initial moisture content $\theta_i$=0.325(cm$^3$/cm$^3$). The calculated hydraulic profiles are confronted to the measured ones under the same boundary and initial conditions by obtained by Bentoumi [14, 15] as shown on figure 5.

The infiltration test was carried out on wet initial state close to saturation $\theta_i$=0.325 corresponding to the degree of saturation about $S_r$=93.39%, so the swelling potential is relatively low. And with no measurement of swelling soil during wetting path by Bentoumi [14, 15], the effect of change volume it is not takes account in this study.

From figure 5, we can observe that the calculated hydraulic profiles of Gault clay are in very ahead with respect to the measured ones. When wetting front $Z_f$ at time 7.44 days is 10.5 cm obtained by the computed profile, it is equal to 4 cm from the measured one. So the infiltration estimated is faster than the measured one. We can deduce for the Gault clay that the parameters value of the hydraulic properties adjusted from the retention curve only, can't describe the hydraulic conductivity curve, and also the spatio-temporal variation of the moisture in the soil θ (z,t).

### 4.2 Givors silt

The simulation was carried out on a 25 cm long soil column. A zero water head pressure is imposed at the top of the column combined with a zero flux at the bottom of the column. The initial water content value is $\theta_i$=0.215(cm$^3$/cm$^3$), the same as the one used in experimental tests. van Genuchten-Mualem empirical model is used, with the parameters values listed in table 2 and the calculated value of $K_s$ (table 8). The results are shown in Figure 6.

Figure 6 shows that the computed profiles of Givors silt are close to the measured ones, especially at the time

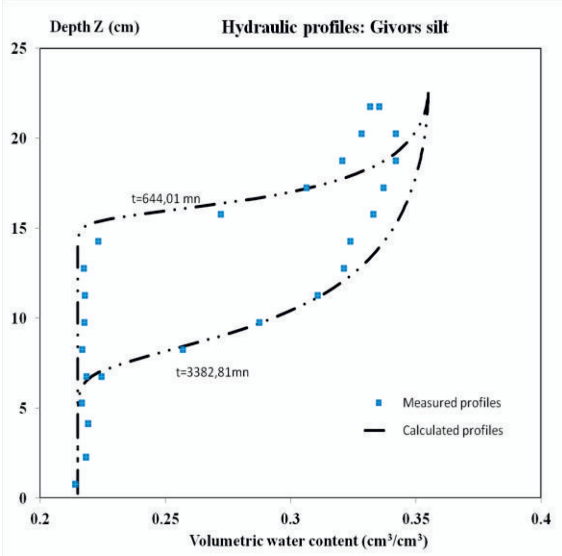


**Figure 6.** Hydraulic profiles of Givors silt

3382.81 mn. A slight difference of surface saturation is found. So we can deduce for the Givors silt, that the parameters value of the hydraulic properties adjusted from the retention curve can describe the hydraulic conductivity curve with a slight correction of the value of $K_s$. Therefore the spatio-temporal variation of the moisture in the soil θ (z,t).

## 5 Conclusions

In this study, we have shown that the parameters adjustment of the empirical models describing the hydraulic properties (retention curve and hydraulic conductivity), from the measured points of the retention curve, does not necessarily lead to well describe the curve hydraulic conductivity, and provide the progress of the moisture front presented by the water profile (case of the Gault clay).

We have also shown that some corrections made on the hydraulic conductivity at saturation ( knowing that the value of the latter is vitiated by the errors) with keeping the adjusted parameters from the retention curve only, can lead to the good description of the hydraulic conductivity curve and the hydraulic profiles also (case of the Givors silt).

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