DEVELOPMENT OF A COMBINED SCHEME: MIXED HYBRID FINITE ELEMENT-FINITE VOLUME (MHFE-FV) FOR MODELING OF CONTAMINED TRANSPORT IN UNSATURATED POROUS MEDIA

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Abstract
This work is devoted to developing numerical method for simulating flow and solute transport in porous media with multiphase taking tale of interaction solid/solute. In more precise, the problem studied is modeled by a coupled system consisting of an elliptical equation (for drainage), and an equation like convection-diffusion-reaction (for transfers). Numerical simulations were realistic for the two-dimensional problems confirm the stability and efficiency of the combined scheme, in the characterization of pollutant transport through the unsaturated zone of an industrial site.

Introduction

In the classical theory of drainage, the flow is assumed to be vertical in the unsaturated zone (USZ) and horizontal in the groundwater. This is the concept of the mixed layer. However, many authors (Vauclin et al, 1979, Clement et al 1996, Romano et al 1999, Kao et al, 2001; Silliman et al, 2002) emphasize the existence of a significant part of the infiltration horizontal elapsing in this zone. The importance of the unsaturated zone as an integral part of the hydrological cycle has long been recognized. The zone plays an inextricable role in many aspects of hydrology, including infiltration, soil moisture storage, evaporation, plant water uptake, groundwater recharge, runoff and erosion. Interest in the unsaturated zone has dramatically increased in recent years because of growing concern that the quality of the subsurface environment is being adversely affected by agricultural, industrial and municipal activities. In these areas the calculated and numerical simulation are essential because the experiments are very difficult if not impossible, by cons the predictions are vital.

The application of the classical conforming finite element method to fluid flow in porous media, in general, does not locally conserve mass [20] due to its global continuity requirement. Many specialized methods have been developed to resolve these difficulties. Eulerian methods use fixed spatial grids and incorporate some form of upstream weighting or other dissipation techniques in their formulations, [06], [18] and the references therein). Hence, they can eliminate the nonphysical oscillations present in the standard finite difference and finite element methods. Some of the Eulerian methods, such as the Godunov scheme, the total variation diminishing (TVD) schemes, and the ENO/WENO schemes, can resolve shock discontinuities from the nonlinear hyperbolic conservation laws. Characteristic methods take advantage of the hyperbolic nature of the convection-diffusion equations by utilizing a characteristic tracking to treat the advective component of the governing equation [08].These methods symmetrize the governing equations and significantly reduce the temporal truncation errors. Thus, they allow large time steps to be used in numerical simulations without loss of accuracy, and lead to a greatly improved efficiency. However, most characteristic methods have difficulties in treating flux boundary conditions when a characteristic is tracked to the boundary of the domain, and fail to conserve mass.

Finite volume (FV) methods are a class of discretization schemes that have proven highly successful in approximating the solution of a wide variety of conservation laws systems. There is an extensive literature on this subject. We will not attempt a literature review here, but merely mention a few references. They are extensively used in fluid mechanics, meteorology, electromagnetics, semi-conductor device simulation, models of biological processes and many other engineering areas governed by conservative systems that can be written in integral control volume form, [03], [07], [09], [11], [14], [15], [16]. The primary advantages of these methods are numerical robustness through the obtention of discrete maximum principles, applicability on very general unstructured meshes, and the intrinsic local conservation properties of the resulting schemes. The literature associated with the foundation and analysis of the finite volume methods for hyperbolic problems is extensive, [09], [14] and the references therein). FV methods for elliptic problems have been proposed and analyzed under a variety of different names: box methods, covolume methods, diamond cell methods, integral finite difference methods and finite volume element methods (see, e.g., [15] for a review).

More recently, FV methods were developed and analyzed for convection-diffusion problems (see, for instance, [01], [02], [05], [07], [09], [10], [12], [13], [17], and [19]). There are various approaches in deriving FV approximations of convection-diffusion equations. The most general classification is obtained depending on the choice of: (1) the finite volumes and (2) the discrete space to which the approximate solution belongs. The domain is meshed and depending on whether the finite volumes are the elements from the original splitting or volumes around the vertices of the original splitting, we have correspondingly cell-centered and vertex-centered finite volume methods. For the vertex-centered finite volumes, depending on whether the discrete space is piecewise constant over the finite volumes or piecewise linear over the original mesh, we have correspondingly vertex-centered finite volume difference methods or vertex- centered finite volume element methods. The cell-centered finite volumes can lead to cell-centered finite volume difference methods or mixed methods.

In the first part of this work, we present a mathematical model that describes the flow and the solute transport through variably saturated porous media and multiphase, taking into account the interaction between the solid phase and liquid phase (effect of delay, degradation, and adsorption). The second part is devoted to analysis and numerical solution of the problem addressed in the first part, by using a combined diagram of mixed finite element and finite volume. The mixed finite element method is used for discretize the flow equation and the scheme of the finite volume is used to approximate the equation of convection-diffusion-reaction. For this we have developed using the computer code FreeFem++[04] a calculation program allowing therefore to solve a coupled system consisting of an elliptical equation discretized by the finite element method mixed hybrid and a linear parabolic equation type convection-diffusion-reaction, discretized by finite volume method type "vertex centered" semi-implicit in time. Finally, in the last section a numerical simulations are performed to verify the one hand the validity of our calculation scheme, and secondly to test the computational efficiency and the prediction of our model on a real example (case of the factory paint the town of Souk AHRAS, ALGERIA).

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