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Explicit solutions for turbulent flow friction factor: A review, assessment and approaches classification

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ABSTRACT

The Colebrook –white equation is widely used in many fields, like civil engineering for calculation of water distribution systems and in all fields of engineering where fluid flow can be occurred. Numerous formulas have been proposed since 1947 in order to simplify the computation of the friction factor, to avoid the iterative procedures methods and to alter the Colebrook-white equation in practice. most of the existing explicit formulas for computation of the friction factor for turbulent flow in rough pipes proposed are cited, where thirty three "33" equations have been inventoried. The goal of this paper is to assess the accuracy of each model and to propose an arrangement from the best to the lower accuracy according to a proposed method combined of three criteria which are: simplicity of the formula, maximum deviations and the coverage of the entire range of Moody diagram.

1. Introduction

Estimating head loss is an important task in the hydraulic engineer's life practice. Water supply network is prime example, where the implicit Colebrook–White equation has been widely used to estimate the friction factor for turbulent fluid-flow in Darcy–Weisbach equation. It is therefore no surprise that it has attracted a lot of attention by both practitioners and researchers over the past years. The Darcy–Weisbach model for steady, uniformly distributed head losses reported in Eq. (1) probably represents the most well-known formula where the friction factor "f" is used to compute the slope hydraulic grade line J (i.e. the head loss per unit length of a pipe [1]:

$$\mathbf{j} = \frac{\mathbf{8fL}}{\pi^2\mathbf{gD}^5}\mathbf{Q}^2 \tag{1}$$

where

j: the hydraulic energy slope (m),
f: friction factor,
L: length of the pipe (m),
D: Intern pipe diameter (m)
Q: Flow (m^3/s)

To solve the Eq. (1), the iterative procedure is necessary and inevitable for the first time, which involve evidently a huge time for the computation of the friction factor and by the way the computation of all the physical system where the friction values makes part.

The flow is qualified turbulent if the Reynolds number is equal or exceed 2300 [2]:

$$R_e \geq 2300 \tag{2}$$

According to Moody's diagram four regions can be defined: laminar region where the roughness has no discernible effect; transition region is an intermediate region between the smooth and rough zone, the friction factor values depend on the relative roughness of the pipe and $\mathbf{R_e}$; the hydraulic smooth flow regime known by the moderate degree of roughness, the pipe acts as a smooth pipe; and the full developed turbulence region where f is independent of $\mathbf{R_e}$ [3,4].

For the laminar region, the fiction factor depends only on Reynolds number:

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$$f = \frac{64}{R_e} \tag{3}$$

In the full developed turbulence region the friction coefficient depends only on relative roughness, Nikuradse's turbulent pipe flow investigations are achieved to the following formula [5,6]:

$$f = \left[-2log\left(\frac{\varepsilon/D}{3.7}\right)\right]^{-2} \tag{4}$$

where

ε: is the average roughness height (Or, the equivalent Nikuradse's sand-grain roughness),
D: is the interne pipe diameter.

The computation of the friction factor is explicit for laminar and developed turbulence.

Before 1939 when Colebrook-White equation was published for turbulent regime in smooth pipes (The fourth region) the implicit Prandtl's equation was widely used. Prandtl derived a formula from the logarithmic velocity profile and available experimental data on smooth pipes [7]; which is known as Prandtl-Von Karman equation [7,8].

$$f = \left[-2log\left(\frac{2.51}{R_e\sqrt{f}}\right)\right]^{-2} \tag{5}$$

In 1937 Colebrook according to experiments data conducted by himself, his colleague White established a curve fit and presented the well-known formula Colebrook-White [9]; which is the first equation allow to describe the flow in the transition region, hence this formula is consisted of two terms presented by Nikuradse and Prandtl:

$$f = \left[-2log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{R_e\sqrt{f}}\right)\right]^{-2} \tag{6}$$

All cited authors were trying to approximate the White-Colebrook equation by fitting the points that are given in Moody's diagram. Moody's diagram was in fact formed from data obtained by using the White-Colebrook formula which is in fact an approximation of the Nikuradse's harp and not a too successful one (which is widely recognized) due to high degree of inconsistence with the original in the region of transition to turbulence [7,1].

Sletfjerding et al. [10] reported that Nikuradse used sand-grains and "Japanese lacquer" to vary the surface roughness of his test pipes. Several authors have questioned Nikuradse's measurements. Zagarola [11] gives a critical review of the smooth pipe measurement by Nikuradse [12] (the experimental set-up for Nikuradse's rough pipe experiments was similar to the smooth pipe set-up). Zagarola noted several inconsistencies in Nikuradse's experimental set-up and work. Grigson [13] showed how problems in defining the origin for the logarithmic velocity profile makes it difficult to determine the Von Karman constant from Nikuradse's measurements.

Using the data extracted from tables 2–7 of Nikuradse [12] for turbulent flow, several authors [7,14,15] confirm the fact that at the transitional regime between the smooth and rough flow is not comply with Colebrook white results. Most friction factor correlations used in industry are semi-empirical models based on turbulent boundary layer theory. The difference between the roughness of the commercial pipes and the sand used by Nikuradse [16] is the main cause for the difference between the two formulas of Colebrook-white and Nikuradse.

It is not easy to calculate the friction factor "f" using Eq. (6). The friction factor appears in the both sides, the Colebrook's equation is implicit, and it has to be solved iteratively which causes serious difficulties in repetitive calculations of the friction factor. For this reason, a number of approximate solutions have been proposed to alter the Colebrook's equation; thirty three (33) equations were inventoried. Several authors have reviewed these equations by looking at the accuracy only [17–19]. The More the equation is longer the more the CPU time is higher. However the explicit formulas which are the topic of the paper the time need for their computation is "zero" so the only difference that can make sense is the environment and the work size in which the use of those formulas is an obligation. The basic idea of this paper is to review most of the existing explicit formulas for computation of the friction factor for turbulent flow in rough pipes and to introduce more than one criterion to select the best equations. Three criteria were proposed: simplicity of the formula, maximum deviations and the coverage of the entire range of Moody diagram.

2. Approximate solutions

The Eq. (6) is implicit which needs for the trial error methods or a graphical solution, the Moody's diagram is surprisingly a good solution for Colebrook-white equation. In general the graphical solutions are not accurate and are limited. For this reason many authors have proposed approximate solutions [20] for Colebrook-White equations [21,22] from 1947 until nowadays which can be presented as following:

2.1. Moody's formula (1947)

In 1947 Moody [23] has proposed a new form of Colebrook-white equation. Moody is widely known by his diagram but not by this proposal to replace the implicit equation which is the oldest explicit proposal:

$$f = 0.0055\left[1 + \left(2\times10^4(\varepsilon/D) + \left(10^6/R_e\right)\right)^{1/3}\right] \tag{7}$$

Moody proposed to use the Eq. (7) for the following range:

$$4000 \le R_e \le 10^8 \text{ and } 0 \le (\varepsilon/D) \le 10^{-2}$$

2.2. Altshul correlation (1952)

Altshul in 1952 - equation from Russian practice - which is cited by Genić et al. [18] and Mustafa et al. [19] gave a friction factor correlation presented as in the Eq. (8):

$$f = 0.11\left(\frac{68}{R_e} + \varepsilon/D\right)^{0.25} \tag{8}$$

According to Round [24] Altshul proposed another formula which is improved from the equation of Konakovt's [25] as following:

$$\frac{1}{\sqrt{f}} = 1.8log\left(\frac{R_e}{R_e\frac{\varepsilon}{D} + 7}\right) \tag{9}$$

Altshul recommends the following range:

$$4000 \le R_e \le 10^7,\ and\ 0 \le (\varepsilon/D) \le 10^{-2}$$

2.3. Wood equation (1966)

Wood [26] tried to gives a formula more simple than the Eq. (6). Wood proposed to use the following formula:

$$f = 0.53(\varepsilon/D) + 0.094(\varepsilon/D)^{0.225} + 88(\varepsilon/D)^{0.44}R_e^{-1.62(\varepsilon/D)^{0.134}} \tag{10}$$

Limit:

$$R_e > 4000, \ \ and \ \ 0 \leq (\varepsilon/D) \leq 5 \cdot 10^{-2}$$

2.4. Eck's equation (1973)

The Eck's Eq. [27] is simpler, it was proposed in 1973 and he suggests the following equation:

$$\frac{1}{\sqrt{f}} \cong -2\log\left(\frac{\varepsilon}{3.715D} + \frac{15}{R_e}\right) \tag{11}$$

The recommended range to use the equation was not found [28].

2.5. Swamee and Jain (1976)

This formula has been widely used as the best explicit approximations of the Colebrook–White formula. Even in much software like EPANET [29,30]. The formula can be expressed as follows:

$$f = \left(-2log\left(\frac{\varepsilon/D}{3.7} + \frac{5.74}{{R_e}^{0.9}}\right)\right)^{-2} \tag{12}$$

The authors proposed to use the Eq. (12) in the following range:

$$5000 < R_e < 10^8 \ \text{and} \ 10^{-6} < \varepsilon/D < 10^{-2}$$

2.6. Approximation of Jain (1976)

The Eq. (13) was proposed in 1976 by Jain after succeeded effort under the following form [31]:

$$\frac{1}{\sqrt{f}} \cong -2\log\left(\frac{\varepsilon}{3.715D} + \left(\frac{6.943}{R_e}\right)^{0.9}\right) \tag{13}$$

Jain recommended using the Eq. (13) for the following range:

$$5000 \leq R_e \leq 10^7 \ \text{and} \ 4 \times 10^{-5} \leq \varepsilon/D \leq 5 \times 10^{-2}$$

2.7. Churchill's formula (1977)

To replace the Colebrook-white equation Churchill proposed to use the following expression [32]:

$$\frac{1}{\sqrt{f}} = -2\text{log}\left[\left[\frac{\left(\frac{\varepsilon}{D}\right)}{3.7}\right] + \left(\frac{7}{R_e}\right)^{0.9}\right] \tag{14}$$

The recommended range to use the equation was not found [33].

2.8. Chen's approximation (1979)

Chen has contributed in the explicit of Colebrook-white equation by a long explicit solution [34]:

$$\frac{1}{\sqrt{f}} = -2\text{log}\left(\frac{\left(\frac{\varepsilon}{D}\right)}{3.7065} - \left(\frac{5.0452}{R_e}\right)\log\left[\left[\frac{\left(\frac{\varepsilon}{D}\right)^{1.1098}}{2.8257}\right] + \left(5.8506/{R_e}^{0.8981}\right)\right]\right) \tag{15}$$

Recommended range:

$$4000 \leq R_e \leq 4 \times 10^8, \ \ and \ \ 10^{-7} \leq (\varepsilon/D) \leq 5 \times 10^{-2}$$

2.9. Shacham correlation (1980)

Correlation proposed by Shacham [35] as following:

$$f = \left(-2log\left(\frac{\varepsilon}{3.7D} - \frac{5.02}{R_e}log\left(\frac{\varepsilon}{3.7D} + \frac{14.5}{R_e}\right)\right)\right)^{-2} \tag{16}$$

Limit:

$$4000 \leq R_e \leq 4 \times 10^8, \ \ and \ \ 0 \leq (\varepsilon/D) \leq 5 \times 10^{-2}$$

2.10. Round approximation (1980)

Approximation proposed by Round [24] is relative and simple:

$$\frac{1}{\sqrt{f}} = -1.8log\left[\frac{R_e}{0.135R_e(\varepsilon/D) + 6.5}\right] \tag{17}$$

For the following validity range:

$$4000 \leq R_e \leq 10^7, \ \ and \ \ 0 \leq (\varepsilon/D) \leq 10^{-2}$$

2.11. Barr approximation (1981)

Approximation proposed by Barr [36] does not require internal iterative calculus:

$$\frac{1}{\sqrt{f}} = -2log(\frac{\varepsilon}{3.7D} + \frac{4.518log(R_e/7)}{R_e\left(1 + R_e^{0.52}/\left(29(\varepsilon/D)^{0.7}\right)\right)}) \tag{18}$$

Limit:

$$2300 \leq R_e \leq 10^8, \ \ and \ \ 0 \leq (\varepsilon/D) \leq 5 \times 10^{-2}$$

2.12. Zigrang and Sylvester approximations (1982)

Approximation proposed by Zigrang and Sylvester [37] don't use internal iterative procedure to achieve a good accuracy:

$$\frac{1}{\sqrt{f}} = -2log\left(\frac{\varepsilon}{3.7D} - \frac{5.02}{R_e}log\left(\frac{\varepsilon}{3.7D} - \frac{5.02}{R_e}log\left(\frac{\varepsilon}{3.7D} + \frac{13}{R_e}\right)\right)\right) \tag{19}$$

Zigrang's equation is recommended for the following range:

$$4000 \leq R_e \leq 10^8, \ \ and \ \ 10^{-5} \leq (\varepsilon/D) \leq 5 \times 10^{-2}$$

2.13. Proposal of Haaland (1983)

In 1983 Haaland published his explicit solution for Colebrook-white equation as follows [38]:

$$\frac{1}{\sqrt{f}} = -1.8log\left[\left[\left(\frac{\varepsilon}{D}\right)/37\right]^{1.11} + (6.9/R_e)\right] \tag{20}$$

Where the recommended range is given as:

$$4 \times 10^3 \leq R_e \leq 10^8, \ \ and \ \ 10^{-6} \leq (\varepsilon/D) \leq 5 \times 10^{-2}$$

2.14. Serghides equation (1984)

Serghides equation is an approximation of the implicit Colebrook–White equation. It is valid for all ranges of Reynolds numbers and relative roughness as follows [39]:

$$\frac{1}{\sqrt{f}} = A - \frac{(B-A)^2}{C - 2B + A} \tag{21}$$

$$A = -2\log\left(\frac{\varepsilon/D}{3.7} + \frac{12}{R_e}\right) \tag{22}$$

$$B = -2\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51A}{R_e}\right) \quad (23)$$

$$C = -2\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51B}{R_e}\right) \quad (24)$$

Limit:

$$2300 \le R_e \le 10^8, \ \ and \ \ 10^{-6} \le (\varepsilon/D) \le 5\times10^{-2}$$

2.15. Tsal correlation (1989)

Tsal [40] developed a relationship that is valid for: $4\times10^3 < \mathrm{Re} < 4\times10^8$ and $0 \le (\varepsilon/\mathbf{D}) \le 5\cdot10^{-2}$

$$\beta = 0.11\left(\frac{68}{R_e} + \frac{\varepsilon}{D}\right)^{0.25} \quad (25)$$

In the case when:

$$\beta \ge 0.018,$$

Then:

$$f = \beta \quad (26)$$

The Eq. (26) of Tsal [40] approximation was already proposed by Altshul [18].
Else:

$$f = 0.018 + 0.85\beta \quad (27)$$

2.16. Formula of Manadilli (1997)

In 1977 Manadilli proposed a direct solution to alter the Colebrook-white equation by using the following formula [41]:

$$\frac{1}{\sqrt{f}} = -2log\left[\left[\frac{\left(\frac{\varepsilon}{D}\right)}{37} + \left(\frac{95}{R_e^{\,0.983}}\right) - \left(\frac{96.82}{R_e}\right)\right]\right] \quad (28)$$

Recommended range:

$$5.235\times10^3 \le R_e \le 10^8 0 \le \left(\frac{\varepsilon}{D}\right) \le 5\times10^{-2}$$

2.17. Approximation of Romeo et al. (2002)

Using multiple variables regression method Romeo has proposed the following formula [42]:

$$\frac{1}{\sqrt{f}} = -2log\left(\frac{\varepsilon}{3.7065D} - \left(\frac{5.0272}{R_e}\right)\log\left(\frac{\varepsilon}{3.827D} - \left(\frac{4.567}{R_e}\right)log\left(\left(\frac{\varepsilon}{7.7918D}\right)^{0.9924} + \left(\frac{5.3326}{208.815 + R_e}\right)^{0.9345}\right)\right)\right) \quad (29)$$

Recommended range:

$$3\times10^3 \le R_e \le 1.5\times10^8, \ \ and \ \ 0 \le (\varepsilon/D) \le 5\times10^{-2}$$

2.18. Achour's formula (2002)

In order to simplify the computation of the friction factor for turbulent pressurized flow, Achour propose to use the following formula [43]:

$$f = \left[-2\log\left(\frac{\varepsilon/D}{3.7} + \frac{4.5}{R_e}\log\frac{R_e}{6.97}\right)\right]^{-2} \quad (30)$$

Recommended range:

$$R_e \ge 10^4, \ \ and \ \ 0 \le \varepsilon/D \le 5\times10^{-2}$$

2.19. Equation of Ajinkya (2006)

Using Jean Henri Lambert function or omega function, where is defined as the inverse function of: $\boldsymbol{Z} = \boldsymbol{we^w}$ i.e., $\boldsymbol{w} = \boldsymbol{lambert}(\boldsymbol{Z})$ denoted by W(z) [44], Ajinkya [45] proposed an analytical solution for Colebrook- white equation. The author proposes to use Wilkes's proposal [46] about the expression of Colebrook- White equation as follows:

$$\frac{1}{\sqrt{f}} = -1.7372\left[ln\left(\frac{\varepsilon}{3.7D} + \frac{1.257}{R_e\sqrt{f}}\right)\right] \quad (31)$$

$$\frac{\varepsilon}{3.7D} = a\frac{1.257}{R_e} = b1.7372 = c.$$

If we replace the amount $(1/\sqrt{\mathbf{f}})\mathbf{by}''\mathbf{y}''$ in the Eq. (31) we obtain the following:

$$y = -c[ln(a + by)]. \quad (32)$$

So:

$$a + by = a - bc[ln(a + by)]. \quad (33)$$

Let's to replace: (a + by) by "ζ" after many arrangements we get the following:

$$\frac{\zeta}{bc} + \ln(\zeta) = \frac{a}{bc} \quad (34)$$

Or:

$$\frac{\zeta}{bc}e^{\zeta}bc = \frac{e^{a/bc}}{bc}. \quad (35)$$

Using the Lambert function definition, the Lambert equation can be writing as follow:

$$\frac{\zeta}{bc} = w_0\left(\frac{e^{a/b}}{bc}\right). \quad (36)$$

After a few elementary simplifications the equation can be rewritten as:

$$f = \frac{1}{[c[W_0(e^{a/bc}/bc)] - a/b]^2} \quad (37)$$

Recommended range:

$$10^{-6} \le \varepsilon/D \le 0.05 \ \ et \ \ 4x10^4 \le R_e \le 10^8$$

2.20. Buzzelli approximation (2008)

Buzzelli (2008) proposed the following approximation [47]:

$$\frac{1}{\sqrt{f}} = A - \left(\frac{A + 2log(B/R_e)}{1 + (2.18/B)}\right) \quad (38)$$

where

$$A = \frac{0.7741ln(R_e) - 1.41}{1 + 1.32\sqrt{\varepsilon/D}}$$

$$B = (\varepsilon/3.7D)R_e + 2.51A$$

Recommended limit:

$$2300 \le R_e \le 10^8, \ \ and \ \ 0 \le (\varepsilon/D) \le 5\times10^{-2}$$

2.21. Sonnad – Goudar equation (2006)

Goudar-Sonnad equation is an approximation of the implicit Colebrook–White equation. It has the following form [48]:

$$\frac{1}{\sqrt{f}} = 0.8686ln\left(\frac{0.4587R_e}{S^{\left(\frac{S}{S+1}\right)}}\right) \quad (39)$$

where

$$S = 0.124(\varepsilon/D)R_e + ln(0.4587R_e)$$

Limit:

$$4000 \le R_e \le 10^8, \; and \; 10^{-6} \le (\varepsilon/D) \le 5 \times 10^{-2}$$

2.22. Rao and Kumar approximation (2007)

The Approximation proposed by Rao and Kumar in 2007 is given by the Eq. (40) [49]:

$$\frac{1}{\sqrt{f}} = 2log\left(\frac{(2\varepsilon/D)^{-1}}{((0.444+0.135R_e)/R_e)F(R_e)}\right) \quad (40)$$

where

$$F(R_e) = 1 - 0.55e^{-0.33(ln(R_e/6.5))^2}$$

Limit:

$$2300 \le R_e \le 10^8, \; and \; 10^{-6} \le (\varepsilon/D) \le 5 \times 10^{-2}$$

2.23. Avçi and Karagoz approximation (2009)

The formula proposed by Avçi and Karagoz (2009) has been developed from the experimental Princeton super-pipe data [50]:

$$f = \frac{6.4}{\left(lnR_e - ln\left(1 + 0.01R_e(\varepsilon/D)\left(1 + 10\sqrt{\varepsilon/D}\right)\right)\right)^{2.4}} \quad (41)$$

Limit:

$$2300 \le R_e \le 10^8, \; and \; 0 \le (\varepsilon/D) \le 5 \times 10^{-2}$$

2.24. Vatankhah and kouchakzadeh (2009)

Sonnad and Goudar in 2006 [48] presented a relationship for the friction factor which has been improved by the discussers [51] as following:

$$f = \left(aln\left(d/(S-0.28)^{\frac{S}{S+0.98}}\right)\right)^{-2} \quad (42)$$

where

$$a = 0.8686, \; d = 0.4587R_e, \; and$$
$$S = 0.1240(\varepsilon/D)R_e + ln(0.4587R_e)$$

Limit:

$$10^4 \le R_e \le 10^8, \; and \; 10^{-6} \le (\varepsilon/D) \le 10^{-2}$$

2.25. Papaevangelou correlation (2010)

Papaevangelou et al. [52] developed an explicit formula given by Eq. (43):

$$f = \frac{0.2479 - 0.0000947(7 - logR_e)^4}{\left(log\left(\frac{\varepsilon/D}{3.615} + 7.366/R_e^{0.9142}\right)\right)^2} \quad (43)$$

Limit:

$$4000 \le R_e \le 10^8, \; and \; 10^{-6} \le (\varepsilon/D) \le 5 \times 10^{-2}$$

2.26. Brkic correlation (2011)

Brkic in 2011 [17] developed a relationship to calculate explicitly the friction factor as follows.

$$\frac{1}{\sqrt{f}} = -2log\left(10^{-0.4343\beta} + \varepsilon/3.7D\right) \quad (44)$$

where

$$\beta = ln(1 + 0.458R_e)\left(1 - \frac{ln(1 + ln(1 + 0.458R_e))}{2 + ln(1 + 0.458R_e)}\right) \quad (45)$$

Limit:

$$4000 \le R_e \le 10^8, \; and \; 0 \le (\varepsilon/D) \le 5 \times 10^{-2}$$

2.27. Fang correlation (2011)

Fang et al. [53] developed a relationship that is valid for $3 \times 10^3 < \mathrm{Re} < 4 \times 10^8$, and $0 \le (\frac{\varepsilon}{D}) \le 5 \cdot 10^{-2}$ the formula is presented under the equation (46):

$$f = 1.613\left(ln\left(0.234(\varepsilon/D)^{1.1007} - \left(60.525/R_e^{1.1105}\right) + \left(56.291/R_e^{1.0712}\right)\right)\right)^{-2} \quad (46)$$

Limit:

$$3000 \le R_e \le 10^8, \; and \; 0 \le (\varepsilon/D) \le 5 \times 10^{-2}$$

2.28. Ghanbari–Farshad–Rieke's correlation (2011)

Correlation proposed by Ghanbari et al. [54] is based on data collected from the Moody diagram. The range of applicability of their equation is between the relative roughness of $\varepsilon/D = 0.0$ to 0.05 and the Reynolds numbers ranging from 2100 to 10^8.

$$f = \left(-1.52log\left(\left(\frac{\varepsilon/D}{7.21}\right)^{1.042}\right) + (2.731/R_e)^{0.9152}\right)^{-2.169} \quad (47)$$

Limit:

$$2300 \le R_e \le 10^8, \; and \; 0 \le (\varepsilon/D) \le 5 \times 10^{-2}$$

2.29. Brkić Dejan equation (2011) [55]

Using different solutions of the Lambert W-function, Brkic (2011) [55] propose an explicit form for the Colebrook- white equation as follows:

$$\frac{1}{\sqrt{f}} = -2log\left(\frac{2.18 \times S}{R_e} + \varepsilon/3.71D\right) \quad (48)$$

where "S" is given by the following expression:

$$S = ln(1 + 0.458R_e)\left(1 - \frac{ln(1 + ln(1 + 0.458R_e))}{2 + ln(1 + 0.458R_e)}\right) \quad (49)$$

Limit:

$$2300 \le R_e \le 10^8, \; and \; 0 \le (\varepsilon/D) \le 5 \times 10^{-2}$$

2.30. Formula of Saeed Samadianfard (2012)

Using genetic programming Saeed Samadianfard in 2012 [28] published his work about the explicit computation of the friction factor equation; this later can be expressed as:

$$f = \left(\frac{R_e{}^{\varepsilon/D} - 0.6315093}{R_e{}^{1/3} + R_e \times \varepsilon/D}\right) + 0.0275308\left(\frac{6.929841}{R_e} + \varepsilon/D\right)^{1/9} +$$

$$\left(\frac{10^{\varepsilon/D}}{\varepsilon/D + 4.781616}\right)\left(\sqrt{\varepsilon/D} + \frac{9.997001}{R_e}\right) \quad (50)$$

Recommended range:

$$4x10^3 \le Re \le 10^8 \ \ and \ \ 0 \le \varepsilon/D \le 0.05$$

2.31. Achour approximation (2012)

Using the rough model method - R.M.M – [56] the friction factor can be expressed as following:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{10.04}{\bar{R}}\right) \quad (51)$$

where

$$\bar{R} = 2R_e\left[-\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{5.5}{R_e{}^{0.9}}\right)\right]^{-1} \quad (52)$$

Limit:

$$2300 \le R_e \le 10^8 \ \ and \ \ 0 \le \varepsilon/D \le 0.05$$

2.32. Cojbasic zarko and dejan Brkic (2013) [57]

This proposal is the improvement of Romeo results called model B, which can be written as:

$$\frac{1}{\sqrt{\mathbf{f}}} = -2\mathbf{log}\left((\varepsilon/3.7106\mathbf{D}) - \frac{5}{\mathbf{R_e}}\mathbf{log}((\varepsilon/3.8597\mathbf{D}) - \frac{4.795}{\mathbf{R_e}}\mathbf{log}\left((\varepsilon/7.646\mathbf{D})^{0.9685} + \left(\frac{4.9755}{206.2795+\mathbf{R_e}}\right)^{0.8795}\right)\right)\right) \quad (53)$$

Limit:

$$2300 \le R_e \le 10^8, \ \ and \ \ 0 \le (\varepsilon/D) \le 5 \times 10^{-2}$$

2.33. Ali R. Vatankhah formula (2014) [58]

Using the relative error as an objective function, the approximation of Sonnad and Goudar [48–59] is improved by fine tuning of the coefficient as:

$$f = \left(\frac{2.51/R_e + 1.1513\delta}{\delta - (\varepsilon/D)/3.71 - 2.3026\delta log(\delta)}\right)^2 \quad (54)$$

where

$$\delta = \frac{6.0173}{R_e\left(0.07(\varepsilon/D) + R_e{}^{-0.885}\right)^{0.109}} + \frac{\varepsilon/D}{3.71} \quad (55)$$

Limit:

$$4000 \le R_e \le 10^8, \ \ and \ \ 10^{-6} \le (\varepsilon/D) \le 5 \times 10^{-2}$$

3. Accuracy analysis method

For the best assessment of the reliability of each approximation cited in this paper, many steps ought to be followed and the results will be compared with Colebrook-white relationship.

In this study, we assume that Colebrook-white formula gives the exact values of the friction factor. The comparison will be applied using the following steps:

- Random value of relative roughness is selected from the range:

$$0 \le \frac{\varepsilon}{D} \le 5 \cdot 10^{-2}$$

- Reynolds number belongs for the entire range of Moody diagram :

$$2300 \le R_e \le 10^8$$

- For each value of the relative roughness of the friction factor is calculated using the Colebrook –White formula iteratively for all values of Reynolds number.
- The friction factor for each approximations cited above will be calculated using the appropriate proposed equation.
- The deviation $(\Delta f/f)$ in (%) between the friction factors $f_{proposed}$ and f_{CW} which respectively means the proposed formula and Colebrook-White equation is easily computed using the following formula:

$$\frac{\Delta f}{f} = 100\%\left(\frac{f_{\text{proposed}} - f_{\text{CW}}}{f_{\text{CW}}}\right) \quad (56)$$

- All tests will be dividing in two parts: firstly we start by the proposed range for application of the approximation solutions, and secondly the entire range of Moody. i.e., for :

$$2300 \le R_e \le 10^8 \ \ \text{and} \ \ 0 \le \varepsilon/D \le 5 \cdot 10^{-2}$$

- For the equations where the validity range is not found, the test is applied for the entire range of Moody diagram.

Using the seventh steps cited above, we obtain the following results:

As it can be observed, the obtained results presented in the Table 1 based on the range of applicability proposed by their authors confirm that the explicit formulas are not accuracy and the need for more criteria became necessary, for this reason Table 2 was proposed to select the best equation.

The simplicity of the formula is closely related to the execution time. A simple formula has the shorter CPU time. To assess the simplicity of the equations cited above, Table 3 is established where three criteria are proposed:

- The simplicity of the equation form,
- The length of the equation,
- The number needed to compute the friction factor as proposed by the authors.

To select the best formula three steps were applied:

- The accuracy will be considered as the most important criterion,
- The best formula should be simple as described above,
- In the case of discrepancy of the two above conditions, the accuracy is considered.

4. Discussion

From the above it is clear that the accuracy claimed by many authors was not exact as it is mentioned in the Table 1 and the Fig. 1; 07 formulas cross the sill of 88% such as: Altshul [18], Barr [36], Serghides [39], Tsal [33] and so on, which are to be rejected.

Table 1
Test of the validity range proposed by the authors.

Author	Reynolds number	Relative roughness	$Errmax(\Delta f/f)\%$
Moody	$4000 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 10^{-2}$	$\Delta f/f = 12.08\%$
Altshul	$4000 \le \boldsymbol{R_e} \le 10^7$	$0 \le \boldsymbol{\varepsilon/D} \le 10^{-2}$	$\boldsymbol{\Delta f/f} > 100\%$
Wood	$R_e > 4000$	$0 \le \varepsilon/D \le 5.10^{-2}$	$\Delta f/f = 28.23\%$
Eck	not found	not found	$\Delta f/f = 10.7\%$
Swamee and Jain	$5000 \le R_e \le 10^8$	$10^{-6} \le \varepsilon/D \le 10^{-2}$	$\Delta f/f = 2.81\%$
Jain	$5000 \le R_e \le 10^7$	$4.10^{-5} \le \varepsilon/D \le 5.10^{-2}$	$\Delta f/f = 2.83\%$
Churchill	not found	not found	$\Delta f/f = 4.59\%$
Chen	$4000 \le R_e \le 4 \cdot 10^8$	$10^{-7} \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 36.18\%$
Shacham	$4000 \le R_e \le 4 \cdot 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 0.88\%$
Round	$4000 \le R_e \le 10^7$	$0 \le \varepsilon/D \le 10^{-2}$	$\Delta f/f = 7.85\%$
Barr	$2300 \le \boldsymbol{R_e} \le 10^8$	$0 \le \boldsymbol{\varepsilon/D} \le 5 \cdot 10^{-2}$	$\boldsymbol{\Delta f/f} = 86.95\%$
Zigrang - Sylvester	$4000 \le R_e \le 10^8$	$10^{-5} \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 0.17\%$
Haaland	$4.10^3 \le R_e \le 10^8$	$10^{-6} \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 1.41\%$
Serghides	$2300 \le \boldsymbol{R_e} \le 10^8$	$10^{-6} \le \boldsymbol{\varepsilon/D} \le 5 \cdot 10^{-2}$	$\boldsymbol{\Delta f/f} \gg 100\%$
Tsal	$4.10^3 \le \boldsymbol{R_e} \le 10^8$	$0 \le \boldsymbol{\varepsilon/D} \le 5 \cdot 10^{-2}$	$\boldsymbol{\Delta f/f} > 100\%$
Manadilli	$5.235 \cdot 10^3 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5.10^{-2}$	$\Delta f/f = 2.5\%$
Romeo	$3 \times 10^3 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 0.16\%$
Achour (2002)	$R_e \ge 10^4$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 2\%$
Ajinkya	$4x10^4 \le R_e \le 10^8$	$10^{-6} \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 7\%$
Buzzelli	$2300 \le \boldsymbol{R_e} \le 10^8$	$0 \le \boldsymbol{\varepsilon/D} \le 5 \cdot 10^{-2}$	$\boldsymbol{\Delta f/f} \gg 100\%$
Sonnad	$4000 \le R_e \le 10^8$	$10^{-6} \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 5.32\%$
Rao	$2300 \le R_e \le 10^8$	$10^{-6} \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 8.879\%$
Avçi	$2300 \le \boldsymbol{R_e} \le 10^8$	$0 \le \boldsymbol{\varepsilon/D} \le 5 \cdot 10^{-2}$	$\boldsymbol{\Delta f/f} \gg 100\%$
Vatankhah 2009	$10^4 \le R_e \le 10^8$	$10^{-6} \le \varepsilon/D \le 10^{-2}$	$\Delta f/f = 0.16\%$
Papaevangelou	$4000 \le R_e \le 10^8$	$10^{-6} \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 0.76\%$
Brkić	$4000 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 2.13\%$
Fang	$3000 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 0.54\%$
Ghanbari	$2300 \le \boldsymbol{R_e} \le 10^8$	$0 \le \boldsymbol{\varepsilon/D} \le 5 \cdot 10^{-2}$	$\boldsymbol{\Delta f/f} = 88.02\%$
Brkić 2011a	$2300 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 3.46\%$
Samadianfard	$4000 \le Re \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 6.31\%$
Achour 2012	$2300 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 0.366\%$
Cojbasic	$2300 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 0.18\%$
Vatankhah 2014	$4000 \le R_e \le 10^8$	$10^{-6} \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 0.146\%$

Fig. 1 was drawing using more than five hundred thousand (500,000) values for each equation for deep investigation of the maximum deviation (The other figures were drawing using the same accuracy). Unfortunately the equations of: Serghides [39], Buzzelli [47] and Avçi [50] due to their high values of the maximum relative error ($\Delta f/f \gg 100\%$) their representation becomes useless.

The equations are in general written in simple form except the formulas of: Chen [34], Zigrang [37], Romeo [42], Samadianfard [28] and Cojbasic et al. [57], where they oscillate between two cases: long equations or complicated forms as quoted in the Table 3.

Seven formulas need more than one step to compute the friction factor: Ajinkya [45], Sonnad [48], Vatankhah [51], Brkić [17], Brkić [55], Achour [56] and Vatankhah [58], except the equation of Vatankhah [51] need for three steps to find the value of the friction factor.

For the equation proposed by Moody to avoid the iterative methods according to the accuracy test the error is huge compared with Colebrook-white equation where it exceeded the 10%, the same can be reported for the equations of wood [26], Eck [27], Chen [34]. The Moody equation can be considered as a great result compared to the means of those days (see Fig. 2).

The equations proposed by: Swamee-Jain [29], Jain [31], Churchill [32], Round [24], Manadilli [41], Achour [43], Ajinkya [45], Sonnad [48], Brkiç [17], Brkiç [55], Samadianfard [28] have been enhanced much more than the first group of equations, where the error is reduced to achieve 2% for Achour's formula for example.

Other authors have succeed to get a good approximations even they are not very well known like Haaland [38], where the maximum deviation is almost 1%, this latter is not the only amazing result, where many authors have achieved to formulas with maximum error less than 1% like: Shacham [35], Zigrang and Sylvester [37], Romeo[42], Vatankhah [51], Papaevangelou [52], Fang [53], Achour [56], Cojbasic et al. [57], Vatankhah [58] (see Figs. 3a and 3b).

The most important groups is the latest one, where the maximum deviation is less than 1% for Shacham [35] and Papaevangelou [52] only, but the rest are less than 0.5%. Many of those formulas don't cover the entire range of Moody diagram.

The formulas of: Zigrang and Sylvester [37], Romeo [42], Fang [53], Achour [56], Cojbasic et al. [57], Vatankhah [58] seems cover the almost entire range of Moody diagram.

To refine the list of the best formula, the third criterion of simplicity should be taken into account where according to the Table 3 the formulas of the authors: Zigrang and Sylvester [37], Romeo [42], Vatankhah [51], Cojbasic et al. [57] will be removed from the group.

The formulas of Zigrang and Sylvester [37], Romeo [42] are very accuracy but not simple according to the Table 3, where the discrepancy between the two criterions of simplicity and accuracy was met. According to the selection conditions mentioned above the accuracy is considered first.

Table 2
Test of the entire range of Moody diagram.

Author	Reynolds number	Relative roughness	$Errmax(\Delta f/f)\%$
Moody	$2300 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 16.25\%$
Altshul	$2300 \le \boldsymbol{R_e} \le 10^8$	$0 \le \boldsymbol{\varepsilon/D} \le 5 \cdot 10^{-2}$	$\boldsymbol{\Delta f/f} > 100\%$
Wood	$2300 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 28.24\%$
Eck	$2300 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 10.69\%$
Swamee and Jain	$2300 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 4.52\%$
Jain	$2300 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 4.33\%$
Churchill	$2300 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 4.59\%$
Chen	$2300 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 39.9\%$
Shacham	$2300 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 1.64\%$
Round	$2300 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 8.53\%$
Barr	$2300 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 86.85\%$
Zigrang - Sylvester	$2300 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 0.17\%$
Haaland	$2300 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 2.56\%$
Serghides	$2300 \le \boldsymbol{R_e} \le 10^8$	$0 \le \boldsymbol{\varepsilon/D} \le 5 \cdot 10^{-2}$	$\boldsymbol{\Delta f/f} \gg 100\%$
Tsal	$2300 \le \boldsymbol{R_e} \le 10^8$	$0 \le \boldsymbol{\varepsilon/D} \le 5 \cdot 10^{-2}$	$\boldsymbol{\Delta f/f} > 100\%$
Manadilli	$2300 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 3.31\%$
Romeo	$2300 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 0.16\%$
Achour (2002)	$2300 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 3.06\%$
Ajinkya	$2300 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 7\%$
Buzzelli	$2300 \le \boldsymbol{R_e} \le 10^8$	$0 \le \boldsymbol{\varepsilon/D} \le 5 \cdot 10^{-2}$	$\boldsymbol{\Delta f/f} \gg 100\%$
Sonnad	$2300 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 5.97\%$
Rao	$2300 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 88.79\%$
Avçi	$2300 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f \gg 100\%$
Vatankhah 2009	$2300 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 74.71\%$
Papaevangelou	$2300 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 0.81\%$
Brkić	$2300 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 2.56\%$
Fang	$2300 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 0.57\%$
Ghanbari	$2300 \le \boldsymbol{R_e} \le 10^8$	$0 \le \boldsymbol{\varepsilon/D} \le 5 \cdot 10^{-2}$	$\boldsymbol{\Delta f/f} = 88.02\%$
Brkić [45]	$2300 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 3.46\%$
Samadianfard	$2300 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 8.052\%$
Achour 2012	$2300 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 0.366\%$
Cojbasic	$2300 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 0.175\%$
Vatankhah 2014	$2300 \le R_e \le 10^8$	$0 \le \varepsilon/D \le 5 \cdot 10^{-2}$	$\Delta f/f = 0.146\%$

Table 3
Assessment of the simplicity of the equations.

Author	Simplicity description	Number of computation steps of the friction factor
Moody	Simple	One step
Altshul	**Not considered**	
Wood	Simple	One step
Eck	Simple	One step
Swamee and Jain	Simple	One step
Jain	Simple	One step
Churchill	Simple	One step
Chen	Long and complicated	One step
Shacham	Simple	One step
Round	Simple	One step
Barr	**Not considered**	
Zigrang and Sylvester	Long and complicated	One step
Haaland	Simple	One step
Serghides	**Not considered**	
Tsal	**Not considered**	
Manadilli	Simple	One step
Romeo	Long and complicated	One step
Achour (2002)	Simple	One step
Ajinkya	Simple	two steps
Buzzelli	**Not considered**	
Sonnad	Simple	Two steps
Rao	**Not considered**	
Avçi	**Not considered**	
Vatankhah 2009	Simple	Three steps
Papaevangelou	Simple	One step
Brkić	Simple	Two steps
Fang	Simple	One step
Ghanbari	**Not considered**	
Brkić 2011a	Simple	Two steps
Samadianfard	Long	One step
Achour 2012	Simple	Two steps
Cojbasic	Long and complicated	One step
Vatankhah 2014	Simple	Two steps

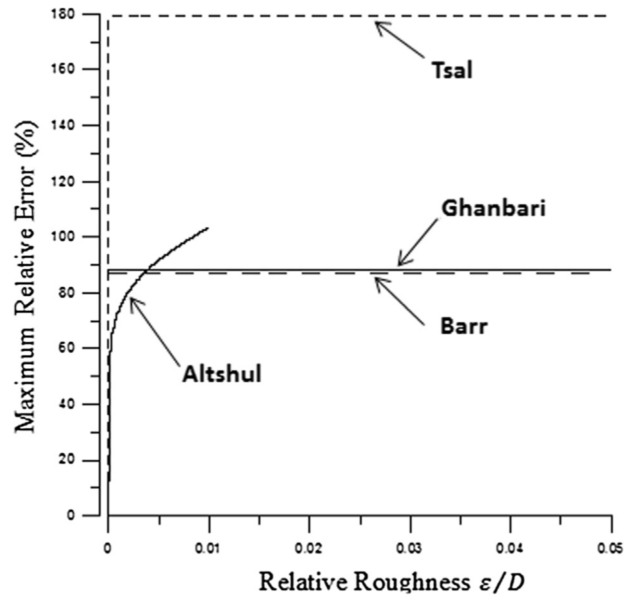

Fig. 1. The maximum deviation (in percent) for versus different relative roughness Values of the rejected formulas.

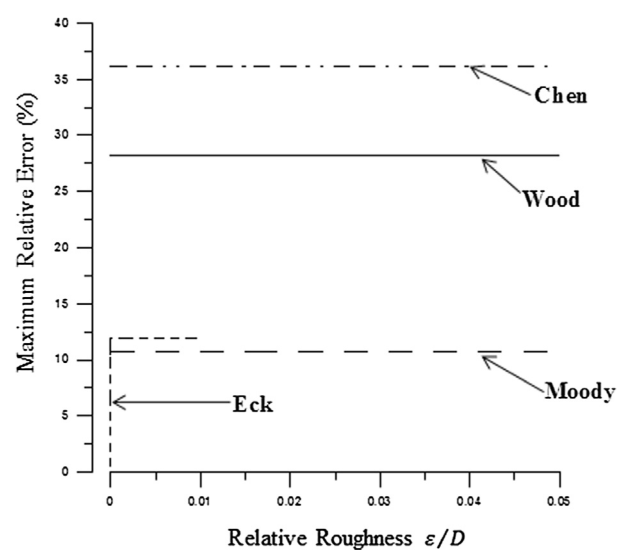

Fig. 2. The maximum deviation (in percent) for versus different relative roughness Values of the Authors: Moody, Wood, Eck, Chen.

Based on the above analysis, it remains in the set only the equations of: Zigrang and Sylvester [37], Shacham [35], Romeo [42], Papaevangelou [52], Fang [53], Achour [56], Vatankhah [58] which they will be considered as the best formulas.

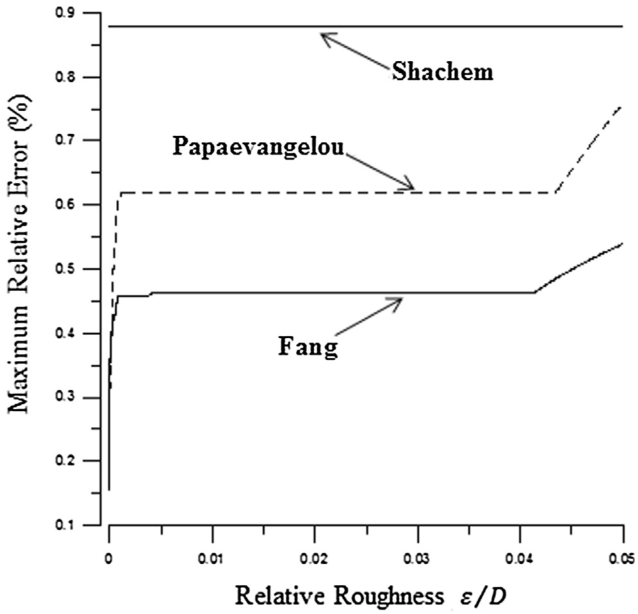

Fig. 3a. The maximum deviation (in percent) for versus different relative roughness Values of the Authors: Shachem, Papaevangelou, Fang.

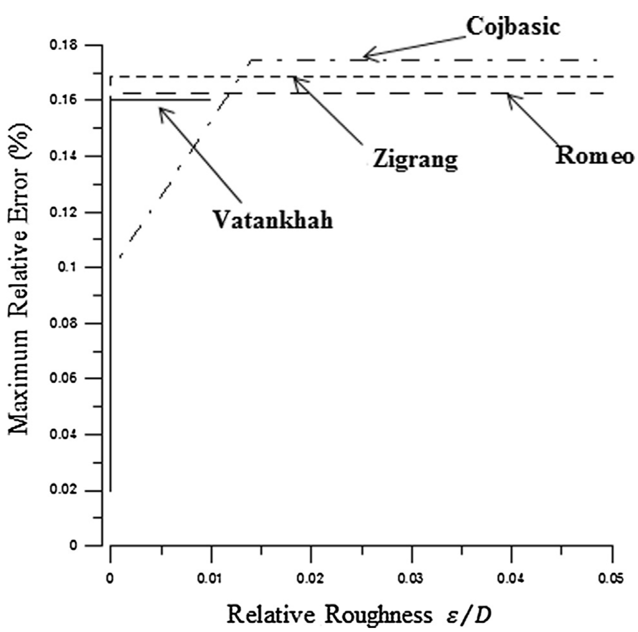

Fig. 3b. The maximum deviation (in percent) for versus different relative roughness Values of the Authors: Zigrang, Romeo, Vatankhah [51], Cojbasic.

From the Tables 1–3 and according to the select conditions of the best formula if the number of the few steps (which oscillate between one and two steps) needed for the computation of the friction factor were neglected, without doubt the best result is recorded for the formula of Vatankhah [58] where the max deviation is 0.146% for the entire range of Moody diagram, in the second position we find Romeo [42] equations with 0.16% as maximum deviation, in the third position we've the formula of Zigrang and Sylvester [37] with 0.17% as a maximum error. The Achour's formula [56] deserves the Fourth position with only 0.366% as max deviation for the entire range of Moody diagram (see Fig. 4).

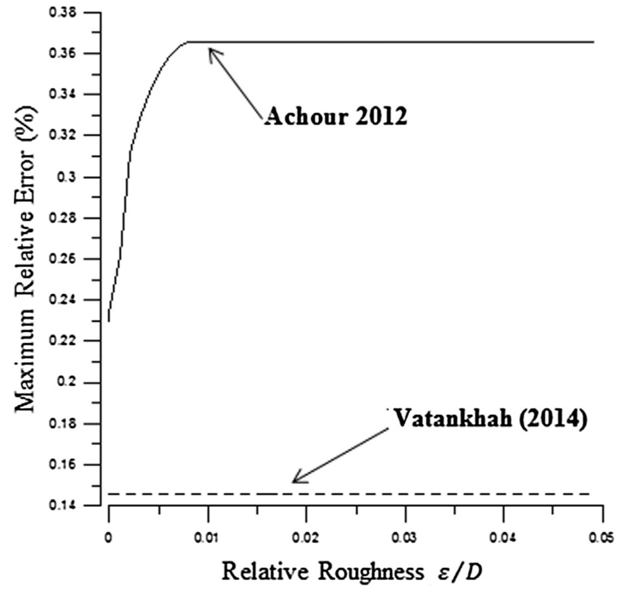

Fig. 4. The maximum deviation (in percent) for versus different relative roughness Values of the formulas: Vatankhah [58] and Achour [56].

5. Conclusions

A state-of-the art review of the most important explicit approximations has been studied. Thirty three (33) formulas were analyzed using the method based on three criteria: Accuracy of the formula, where the Maximum relative error was determined for each approximation, two tests have been applied, firstly on the proposed range of the approximation and the second on the entire range of Moody's diagram. Additional two criteria are proposed: the coverage of the entire range of Moody diagram and the simplicity of the formula. Based on the present comparative analysis, mainly the following findings can be observed:

- Four (04) equations can deserve to be the best approximations, the best one from these four formulas is the equation of Vatankhah [58],
- The formulas of Cojbasic et al. [57] and Fang [53] although they don't belongs to the best group, the maximum errors recorded for both are respectively: 0.18% and 0.54% for the entire range of Moody, they can be considered among the best approximations too.
- There are a lot of accurate explicit approximations but they are not very known, like: Shacham [35], Zigrang and Sylvester [37], Romeo [42], Vatankhah [51], Papaevangelou [52], Fang [53], Achour [56], Cojbasic et al. [57], Vatankhah [58].
- For future researches, it is recommended the utilization and successes of Soft Computing techniques such as: Artificial Neural Network (ANN), Gene Expression Programming (GEP) and so, these tools could strongly contribute to improve results [60–63].

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