Abstract—This paper deals with the problem of state and fault estimation for nonlinear uncertain systems described by Takagi-Sugeno (T-S) fuzzy models. Both measurable and estimated decision variables cases are considered. Indeed a sensor fault detection observer is synthesized with a guaranteed \( \mathcal{L}_2 \) performance to minimize the effect of external disturbance. To achieve this end, a descriptor design approach is used by considering the sensor fault as an auxiliary state variable. Then, stabilization conditions in the sense of Lyapunov method are derived and expressed as a Linear Matrix Inequality (LMI) formulation. Illustrative example is given to show the effectiveness of the given results.

Keywords— Nonlinear systems; Takagi-Sugeno models; state estimation; sensor fault diagnosis; \( \mathcal{L}_2 \) optimization; uncertain systems; descriptor observer.

I. INTRODUCTION

Fault detection, fault diagnosis and fault management play an increasing role for nonlinear systems, in order to detect, isolate and identify sensor, actuator, and systems faults (see [1], [2], [3], [4], [5] and the reference therein). In fact, the increasing demand of reliability, availability and performance of systems has led to the use of nonlinear models to represent the systems. Hence obtained models are very complex and task of model-based fault diagnosis becomes more difficult to achieve.

Recently, nonlinear systems described by T-S models have been considered actively and specially in the fields of control, state estimation and diagnosis of nonlinear systems. The popularity of T-S modeling framework is due, on the one hand, to the property of universal approximation [6] and, on the other hand, to its ease of manipulation from the mathematical point of view compared to the original nonlinear models. Indeed, this approach provides a representation of nonlinear systems by means of interpolating the behavior of several linear submodels which are interconnected by nonlinear functions as a convex combination.

In the field of observer design for fault diagnosis, only a few works exist treating the problem of sensor or actuator faults management when system modeling is subject to uncertainties or time varying parameters. In the literature, the term uncertainty is defined to the model parameters [7], [8], to the model inputs [9], or to the computer implementation [10].

In this paper, the presented work focus on the observer design for T-S models subject to parameter uncertainty, needed to represent with a good precision the system behavior. In this context, very interesting approaches were proposed by assuming the premise variables to be measurable [11], [12]. This assumption allows using separation property, and thus the premise variables of the observer and the model plant can be selected the same [13]. Though, this assumption is very restrictive, since the premise variables depends on the unmeasurable premise variables for a large class of fuzzy systems. Hence, the observer design for T-S models with estimated decision variables has been addressed in some paper. The design of stable filter for T-S fuzzy systems with unmeasurable premise variables has been discussed in [14] by considering systems under perturbed formulation. In [15] robust fault detection design procedure is designed for uncertain continuous-time switched delay systems. The model uncertainties are norm-bounded structure and considered in an additive way. Recent work in [16] focuses on parameter estimation for uncertain T-S systems based on fault diagnosis. The uncertainties are modeled in a polynomial way which allows considering the uncertainty estimation as a fault detection problem. Moreover, a robust sensor fault detection observer attenuating the external disturbances while remaining sensitive to sensor fault design for uncertain discrete-time nonlinear systems is proposed in [17]. The proposed T-S observer is used to estimate jointly states and faults signals by means of a mixed \( H_\infty \) performance index.

The main contributed of this paper is to propose a method to estimate state variables and sensor faults for uncertain nonlinear systems, presented under T-S multi-model formulation. Both measurable and unmeasurable cases are considered. For the state and sensor fault estimation, an idea is to append the sensor fault to the state vector. However conventional observers cannot be used in this case. Hence a descriptor observer is proposed while robustness objective will be taken into account by using \( \mathcal{L}_2 \) approach.

The outline of the paper is as follows. Section 2 recalls the structure of continuous uncertain T-S models. The problem formulation is illustrates by giving some preliminaries on the observer structure and the robustness conditions. In Section 3, a convergent descriptor T-S observer is designed and sufficient conditions are derived in terms of LMIs. The two cases of measurable premise variables and decision variables depending on the state variables estimated by the T-S
observer are considered. A numerical example is provided to illustrate the effectiveness of the proposed approach.

The following notations are considered. \( \mathcal{H}(S) \) denotes the Hermitian of the matrix \( S \), i.e. \( \mathcal{H}(S) = S + S^T \). \( I_n \) is the identity matrix of dimension \( n \times n \) and the symbol * indicates the transposed element in the symmetric positions of a matrix.

II. UNCERTAIN CONTINUOUS T-S MODEL

A. Problem statement

Model uncertainties refer to the difference between the system model and the real one. It can be caused, for instance, by changes within the process itself or in the environment around it. Let us consider the nonlinear dynamic model described by the following if then rules and taking into account the model uncertainties described by an additive perturbation:

Rule i: IF \( \xi_1(t) \) is \( M_{s1} \) and, ..., and \( \xi_g(t) \) is \( M_{si} \), THEN

\[
\begin{align*}
\dot{x}(t) &= (A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t) \\
y(t) &= Cx(t)
\end{align*}
\]

where \( x(t) \in \mathbb{R}^n \) is the state vector, \( u(t) \in \mathbb{R}^m \) is the input vector, and \( y(t) \in \mathbb{R}^p \) is the output vector. \( A_i, B_i, \) and \( C \) are constant real matrices with appropriate dimensions. \( \xi_j (j = 1, ..., g) \) are the premise variables. \( M_{ij} (i = 1, ..., s) \) are the fuzzy sets and \( r \) is the number of rules. Using the interpolation between the local LTI models, the overall T-S continuous model affected by sensor faults \( f_s(t) \) and unknown bounded disturbance \( d(t) \) is inferred as follows:

\[
\begin{align*}
x(t) &= \sum_{i=1}^{r} \mu_i(\xi(t))((A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t) + B_dd(t)) \\
y(t) &= Cx(t) + D_s f_s(t)
\end{align*}
\]

where:

\[
\mu_i(\xi(t)) = \frac{w_i(\xi(t))}{\sum_{i=1}^{r} w_i(\xi(t))}, \quad w_i(\xi(t)) = \prod_{j=1}^{r} M_{ij}(\xi_j(t))
\]

The normalized activation functions \( \mu_i(\xi(t)) \) satisfies

\[
\begin{align*}
0 &\leq \mu_i(\xi(t)) \leq 1 \\
\sum_{i=1}^{r} \mu_i(\xi(t)) &= 1 \quad \forall i \in \{1, 2, ..., r\}
\end{align*}
\]

The normalized activation functions \( \mu_i(\xi(t)) \) satisfies

\[
\begin{align*}
0 &\leq \mu_i(\xi(t)) \leq 1 \\
\sum_{i=1}^{r} \mu_i(\xi(t)) &= 1 \quad \forall i \in \{1, 2, ..., r\}
\end{align*}
\]

In order to estimate the state vector \( x(t) \) and the sensor fault \( f_s(t) \), an augmented system is constructed using the descriptor technique. The main idea is to join the sensor fault to the state vector as an auxiliary state variable. Consequently the uncertain T-S model (2) is rewritten as:

\[
\begin{align*}
\dot{E}\hat{x}(t) &= \sum_{i=1}^{r} \mu_i(\xi(t))(A_i \hat{x}(t) + B_i u(t)) + N h(t) + \hat{B}_d d(t) \\
y(t) &= C E\hat{x}(t) + h(t)
\end{align*}
\]

where

\[
\begin{align*}
h(t) &= D_s f_s(t) \in \mathbb{R}^p, \quad \hat{x}(t) = [x, h] \quad E = \begin{bmatrix} I_n & 0 \\ 0 & 0 \end{bmatrix} \\
A_i &= \begin{bmatrix} A_i + \Delta A_i & 0 \\ 0 & -I_p \end{bmatrix}, \quad B_i = \begin{bmatrix} B_i + \Delta B_i \\ 0 \end{bmatrix} \\
N &= \begin{bmatrix} 0 \\ I_p \end{bmatrix}, \quad C = \begin{bmatrix} C & I_p \end{bmatrix}, \quad \hat{B}_d = \begin{bmatrix} B_d \\ 0 \end{bmatrix}
\end{align*}
\]

In this study, a descriptor observer that uses the input \( u(t) \), the output \( y(t) \) and \( h(t) \) as unknown input is given by the following structure:

\[
\begin{align*}
\dot{E}\hat{z}(t) &= \sum_{i=1}^{r} \mu_i(\xi(t))(K_i \hat{z}(t) + \bar{B}_i u(t)) \\
\hat{x}(t) &= z(t) + L y(t) \\
\hat{y}(t) &= C E\hat{x}(t) = C \hat{x}(t)
\end{align*}
\]

where \( z(t) \in \mathbb{R}^{n+p} \) is an auxiliary state vector of the observer, \( \bar{B}_i = [B_i \ 0]^T \) and \( \hat{x}(t) \in \mathbb{R}^{n+p} \) is the state estimation of the system (2). \( K_i, E \) and \( L \) are the observer gains to be determined.

B. Robustness conditions

In this section, robustness with respect to disturbance is considered by using the \( L_2 \)-gain criterion for the following nominal continuous T-S system.

\[
\begin{align*}
\dot{x}(t) &= \sum_{i=1}^{r} \mu_i(\xi(t))(A_i x(t) + B_i u(t) + B_dd(t)) \\
y(t) &= C x(t) + D_s f_s(t)
\end{align*}
\]

Let us define the error state \( e(t) \) by:

\[
\hat{e}(t) = \hat{x}(t) - \hat{x}(t)
\]

It is well known that the \( L_2 \)-gain from \( d(t) \) to \( e(t) \) is bounded if:
where  
\[ \|d(t)\|_2 \neq 0, \|\hat{d}(t)\|_2 < \gamma^2 \]  \hspace{1cm} (10)  

The goal is to find an admissible observer (7) to minimize \( \gamma \), i.e., an observer that have the best robustness to disturbances \( d(t) \). In the following we give conditions to design the descriptor observer for the continuous T-S system (8).

**Lemma 1.** If there exist positive definite symmetric matrices \( P_1 \) and \( P_2 \), Matrix \( \Xi \), non singular matrix \( M \) and scalar \( \gamma > 0 \) such that the following LMI is satisfied for \( i = 1, \ldots, r \),

\[
\begin{bmatrix}
\mathcal{H}(P_2 A_2) + \mathcal{H}(Z_2 C) + I_n & * & *
\end{bmatrix} \begin{bmatrix}
Z_1^T - P_2 CA_1 - Z_2 C & -Z_2 - Z_2^T & * \\
P_1 & -C^TP_2 & -\gamma I
\end{bmatrix} < 0 \]  \hspace{1cm} (11)  

Then there exist a descriptor observer in the form (7) to asymptotically estimate the state and sensor fault signals for the T-S system (8). The observer parameters are given by:

\[
F_i = \begin{bmatrix}
A_i & 0 \\
-C & -I_p
\end{bmatrix} \hspace{1cm} \text{(12a)}
\]

\[
L = \begin{bmatrix}
0 \\
I_p
\end{bmatrix} \hspace{1cm} \text{(12b)}
\]

\[
E = \begin{bmatrix}
I_n + \Xi C & \Xi \\
MC & M
\end{bmatrix} \hspace{1cm} \text{(12c)}
\]

where

\[
M = (P_2^{-1} Z_2 - CP_1^{-1} Z_1)^{-1}
\]

\[
\Xi = P_1^{-1} Z_2 M
\]

**Proof:** The condition (11) can be easily formulated using the bounded real lemma. See for example [19], [18].

### III. STATE AND SENSOR FAULT OBSERVER

Before we present the proposed strategy, the following lemma is needed to provide LMI conditions.

**Lemma 2** [20]: Consider two real matrices \( P \) and \( Y \) with appropriate dimensions, for any positive scalar \( \delta \) the following inequality is verified:

\[
P^T Y + Y^T P \leq \delta P^T P + \delta^{-1} Y^T Y \quad \delta > 0 \]  \hspace{1cm} (15)

#### A. Case of measurable decision variables

This section is devoted to state and fault estimation for uncertain T-S continuous systems. Based on common quadratic Lyapunov function stability conditions are obtained under LMI constraints. For that purpose, the premise variable \( \xi(t) \) is assumed to be measurable, i.e. \( \xi(t) = u(t) \) or \( \xi(t) = y(t) \). Furthermore, an \( L_2 \) attenuation approach will be proposed to minimize the effect of external disturbance on the state error estimation. In the following result, robustness conditions with respect to disturbances are derived.

**Theorem 1:** The observer (7), estimating the state and sensor fault of the uncertain system (2) and minimizing the \( L_2 \)-gain \( \gamma \) of the known and unknown input on the state estimation error, is obtained by finding symmetric positive definite matrices \( P_1, P_2 \), Matrix \( \Xi \), non singular matrix \( M \), and the positive scalars \( \varepsilon_1 \) and \( \varepsilon_2 \) that minimize the scalar \( \gamma \) under the following LMI constraints

\[
M_i < 0, \hspace{0.1cm} i = 1, \ldots, r \]  \hspace{1cm} (16)

\[
M_i = \begin{bmatrix}
\theta_i^{11} & * & * & * & * & * & * & * & * & *
\theta_i^{12} & -Z_2 - Z_2^T & I & * & * & * & * & * & * & *
B_d P_1 & -C B_d P_2 & -\gamma I & * & * & * & * & * & * & *
0 & 0 & 0 & \theta_i^{33} & * & * & * & * & * & *
0 & 0 & 0 & 0 & \theta_i^{44} & * & * & * & * & *
E_A P_1 & E_A P_2 & 0 & 0 & 0 & -\varepsilon_1 I & * & * & * & *
E_B P_1 & E_B P_2 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_2 I & *
\end{bmatrix} \]  \hspace{1cm} (17)

with

\[
\theta_i^{11} = \mathcal{H}(A_i P_1) + \mathcal{H}(Z_i C) + I \]  \hspace{1cm} (18a)

\[
\theta_i^{12} = Z_1^T - P_2 CA_1 - Z_2 C + I \]  \hspace{1cm} (18b)

\[
\theta_i^{33} = -\gamma I + \varepsilon_1 (A_i A_i^T + C A_i A_i^T C^T) \]  \hspace{1cm} (18c)

\[
\theta_i^{44} = -\gamma I + \varepsilon_2 (B B_i^T + C B_i B_i^T) \]  \hspace{1cm} (18d)

The observer gains are then obtained by (12)-(14).

**Proof:** Consider the matrices \( F_i \in \mathbb{R}^{(n+p),(n+p)} \), \( L \in \mathbb{R}^{n \times p} \) and \( E \in \mathbb{R}^{(n+p),(n+p)} \) defined by (12a)-(12c) where \( \Xi \in \mathbb{R}^{n \times p} \) and \( M \in \mathbb{R}^{p \times p} \) chosen non singular. The augmented state estimation obeys to the following nonlinear system:

\[
\hat{E}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))F_i \hat{e}(t) + Q_i \omega(t) \]  \hspace{1cm} (19)

where:

\[
Q_i = [\bar{B}_d \quad \Delta \bar{A}_i \quad \Delta \bar{B}_i]
\]

\[
\omega(t) = [d(t) \quad \bar{x}(t) \quad u(t)]^T
\]

Therefore (19) is equivalent to:

\[
\dot{\hat{e}}(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) S_i \hat{e}(t) + \Xi_i \omega(t)
\]

where \( S_i = E^{-1} F_i \), and \( \Xi_i = E^{-1} Q_i \). Let \( V(\hat{e}(t)) \) denotes the following candidate Lyapunov function

\[
V(\hat{e}(t)) = \hat{e}(t)^TP\hat{e}(t)
\]

where \( P = P^T > 0 \). By respecting the criterion (10), i.e. \( \|\hat{e}(t)\|_2 \leq \gamma \|\omega(t)\|_2 \) the \( L_2 \)-gain from \( \omega(t) \) to \( \hat{e}(t) \) is bounded by \( \gamma \).
Considering the Lyapunov function (23) and the trajectory of \( \hat{e}(t) \) defined by (22), the inequality (24) can be written as:
\[
\sum_{i=1}^{r} \mu_i(\xi(t)) \begin{bmatrix} \hat{e}(t)^T \omega(t) \\ \omega(t) \end{bmatrix} \begin{bmatrix} \mathcal{H}(P_{S_i}) + I & P_{S_i} \tilde{e}(t) \\ \tilde{e}(t)^T P_{S_i} & -\gamma^2 I \end{bmatrix} \begin{bmatrix} \hat{e}(t) \\ \omega(t) \end{bmatrix} < 0
\]
From which we deduce that
\[
\begin{bmatrix}
\theta^{11} & * & * & * & * \\
\theta^{12} & -Z_2 - Z_2^T + I & * & * & * \\
B_dP_1 & -CB_dP_2 & -\gamma^2 I & * & * \\
\Delta A_i(t)P_1 & -CA_i(t)P_2 & 0 & -\gamma^2 I & * \\
\Delta B_i(t)P_1 & -\Delta B_i(t)P_2 & 0 & 0 & -\gamma^2 I \\
\end{bmatrix} < 0
\] (26)
where \( \theta^{11} \) and \( \theta^{12} \) are defined by (18a) and (18b). Isolating the time varying entries \( \Delta A_i(t) \) and \( \Delta B_i(t) \), (26) becomes
\[
\begin{bmatrix}
\theta^{11} & * & * & * & * \\
\theta^{12} & -Z_2 - Z_2^T + I & * & * & * \\
B_dP_1 & -CB_dP_2 & -\gamma^2 I & * & * \\
0 & 0 & 0 & -\gamma^2 I & * \\
0 & 0 & 0 & 0 & -\gamma^2 I \\
\end{bmatrix} < 0
\] (27)
Using lemma 2, and the property of \( \Sigma_d(t) \) and \( \Sigma_d(t) \) given by (4), it can be stated that the LMI (27) is satisfied if the following holds
\[
\begin{bmatrix}
\theta^{11} & * & * & * & * \\
\theta^{12} & -Z_2 - Z_2^T + I & * & * & * \\
B_dP_1 & -CB_dP_2 & -\gamma^2 I & * & * \\
0 & 0 & 0 & -\gamma^2 I & * \\
0 & 0 & 0 & 0 & -\gamma^2 I \\
\end{bmatrix} < 0
\] (28)
With some Schur complements and defining \( \bar{y} = \gamma^2 \), the previous inequality becomes
\[
\sum_{i=1}^{r} \mu_i(\xi(t))M_i < 0
\] (29)
It follows that (24) is satisfied if the LMI (16) holds, which achieves the proof.

A. Case of estimated decision variables

All the decision variables of the uncertain continuous T-S system (2) are assumed to be measurable in the above parts. However, in general, this assumption is not verified. In the following part, we assume that the decision variables depend on states variables estimated by a T-S observer which represents a more class of nonlinear systems. Therefore the activation functions of the observer are different from the activation functions of the uncertain T-S model (2). Hence, the T-S descriptor observer (7) becomes
\[
\begin{align*}
E\hat{z}(t) &= \sum_{j=1}^{r} \mu_j(\hat{\xi}(t)) \left( K_j z(t) + B_j u(t) \right) \\
\hat{x}(t) &= z(t) + Ly(t) \\
\hat{y}(t) &= C_0 \hat{x}(t) = C\hat{x}(t)
\end{align*}
\] (30)
where \( \hat{\xi}(t) \) is the vector of the estimated decision variables depending on the estimated state variables \( \hat{x}(t) \). The augmented system (22) becomes in this case
\[
\begin{align*}
\hat{\xi}(t) &= \sum_{i=1}^{r} \mu_i(\hat{\xi}(t)) \hat{S}_i \hat{e}(t) + \hat{S}_i \hat{o}(t) \\
\hat{y}(t) &= \hat{S}_o \hat{x}(t) = C\hat{x}(t)
\end{align*}
\] (31)
with
\[
\begin{align*}
\hat{\xi}(t) &= \sum_{i=1}^{r} \mu_i(\hat{x}(t)) \left( \left( A_i + \Delta A_i \right) x(t) \right) \\
&+ \left( B_i + \Delta B_i \right) u(t)
\end{align*}
\] (34)
The asymptotic stability of the augmented system (31) can be derived easily as follows.

Theorem 2: The observer (30), estimating the state and sensor fault of the uncertain system (2) and minimizing the \( L_2 \)-gain \( \gamma \) of the known and unknown input on the state estimation error, is obtained by finding symmetric positive definite matrices \( P_1, P_2 \), Matrix \( \mathcal{E} \), non singular matrix \( M \),
with disturbances is defined by the following matrices:

\[ M_i \begin{bmatrix} \theta_{i1}^1 & * & * & * & * & * \\ \theta_{i1}^2 & -Z_2 - Z_2^T + l & * & * & * & * \\ B_i P_1 & -C B_i P_2 & -\gamma l & * & * & * \\ P_1 & -C P_2 & 0 & -\gamma l & * & * \\ 0 & 0 & 0 & 0 & \theta_i^{13} & * \\ 0 & 0 & 0 & 0 & \theta_i^{14} & * \\ E_i P_1 & E_i P_2 & 0 & 0 & 0 & 0 - \varepsilon_i^1 l \\ E_i P_1 & E_i P_2 & 0 & 0 & 0 & 0 - \varepsilon_i^2 l \end{bmatrix} \]

where \( \theta_{i1}^1, \theta_{i1}^2, \theta_i^{13} \) and \( \theta_i^{14} \) are defined by (18a)-(18c). The observer gains are then obtained by (12)-(14).

Proof: The proof is similar to the proof of Theorem 1 and is omitted for the sake of brevity.

IV. SIMULATION EXAMPLE

In this section, the effectiveness of the robust descriptor observer design approach is illustrated on a reduced bioreactor model. This system is widely used in wastewater treatment plant. The state estimation problem is then an important task in monitoring the operation of the process in order to respond to a failure. The bioreactor under consideration can be represented by the following nonlinear system:

\[
\begin{align*}
x_1(t) &= \frac{a x_1(t) x_2(t)}{x_2(t) + b} - x_1(t) u(t) \\
x_2(t) &= -\frac{c a x_1(t) x_2(t)}{x_2(t) + b} + (d - x_2(t)) u(t)
\end{align*}
\]

where \( x_1(t) \) represents the biomass concentration, \( x_2(t) \) is the substrate concentration, \( u(t) \) is the dilution rate. The following parameters are given: \( a = 0.5, b = 0.07, c = 0.7, d = 2.5 \). Using the well-known sector nonlinearity approach [24], a T-S model structure is obtained where the input and state vectors are considered as premise variables and denoted \( \xi_j(.) (j = 1, \ldots, q) \). For \( q \) premise variables, \( r = 2^q \) submodels will be obtained. The above model is constituted by three nonlinearities:

\[
\begin{align*}
\xi_1(t) &= -u(t) \\
\xi_2(x) &= -\frac{a x_1(t)}{x_3(t) + b} \\
\xi_3(x) &= -\frac{c a x_1(t)}{x_3(t) + b}
\end{align*}
\]

A T-S model with eight submodels is then obtained in a compact state space leading to define the intervals variations of \( \xi_1(t), \xi_2(x) \) and \( \xi_3(x) \) by: \( \xi_1(t) \in [-1, -0.2], \xi_2(t) \in [0.004, 15], \xi_3(t) \in [-1.72, -0.2] \). The derived T-S model with disturbances is defined by the following matrices:

\[
A_1 = \begin{bmatrix} -0.2 & 15 \\ -0.2 & -0.2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.2 & 15 \\ -1.72 & -0.2 \end{bmatrix} \\
A_3 = \begin{bmatrix} -0.2 & 0.004 \\ -0.2 & -0.2 \end{bmatrix}, \quad A_4 = \begin{bmatrix} -0.2 & 0.004 \\ -1.72 & -0.2 \end{bmatrix} \\
A_5 = \begin{bmatrix} -1 & 15 \\ -0.2 & -1 \end{bmatrix}, \quad A_6 = \begin{bmatrix} -1 & 15 \\ -1.72 & -1 \end{bmatrix} \\
A_7 = \begin{bmatrix} -1 & 0.004 \\ -0.2 & -1 \end{bmatrix}, \quad A_8 = \begin{bmatrix} -1 & 0.004 \\ -1.72 & -1 \end{bmatrix} \\
B_1 = B = \begin{bmatrix} 0 & 0.37 \\ 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad D_s = \begin{bmatrix} 4 & 0 \\ 0 & 8 \end{bmatrix}, \quad B_d = \begin{bmatrix} 0 & 0.5 \end{bmatrix}
\]

In order to demonstrate the results provided by the proposed approach, we will introduce modeling bounded uncertainties, affecting the parameters \( a, b \) and \( c \). A comparison between the uncertain system and the nominal one is given in Fig.1. The uncertainties are common to the eight sub-models and have the general form:

\[
\Delta A_i(t) = \Delta A(t) = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix} \Sigma_d(t)(1 & 1)
\]

This model of bounded uncertainties represent a large class of modeling uncertainties (dynamic neglected, noisy measurement...). Let us consider the following fault signal \( f(t) = (f_1(t) f_2(t))^T \) affecting the system behavior and described as follows:

\[
f_1(t) = 0.3 \sin(0.1 t) \quad \text{occurs at } 3 \text{sec} \leq t \leq 25 \text{sec}
\]

\[
f_2(t) = \begin{cases} 0.01(t - 5) + 0.2 & 40 \text{sec} \leq t < 60 \text{sec} \\ 0.01 t & 60 \text{sec} \leq t \leq 80 \text{sec} \\ 0 & \text{otherwise} \end{cases}
\]

An unknown disturbance \( d(t) \) with band-limited white noise as given by fig.2 is considered. Applying Theorem 2, the observer (30) is designed by finding positive scalars \( \varepsilon_{1r}, \varepsilon_{2l} \), positive defined matrices \( P_1 \) and \( P_2 \) that are not given here – such that the convergence conditions given in theorem 2 hold. The value of the attenuation rate is \( \gamma = 0.8 \). As illustrates in figures 3, 4 the proposed method gives good estimation of both state variables and sensor faults.
The observer gains matrices are obtained by resolution of the LMIs deriving from the optimization problem in theorem 2:

\[
E = \begin{bmatrix}
0.4121 & 0.1501 & 0.1501 & -0.5879 \\
-0.0983 & 1.1041 & 0.1041 & -0.0983 \\
-0.1214 & 0.0484 & 0.0484 & -0.1214 \\
-0.0376 & -0.0754 & -0.0754 & -0.0376 \\
\end{bmatrix}
\]

\[
M = \begin{bmatrix}
0.0484 & -0.1214 & 0.1501 & -0.5879 \\
-0.0754 & -0.0376 & 0.1041 & -0.0983 \\
\end{bmatrix}
\]

V. CONCLUSION

In this paper, a robust fault detection observer has been designed for uncertain continuous T-S models for both measurable and estimated decision variables. Robustness with regard to external disturbances is realized through \( L_2 \) optimization approach. Sufficient conditions for the existence of such observer are given in terms of linear matrix inequalities. At last a bioreactor model is used to illustrate the effectiveness of the proposed approach.

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